THE LATERAL TORSIONAL BUCKLING STRENGTH OF STEEL I-GIRDERS WITH CORRUGATED WEBS

by

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Abstract

This report addresses the lateral torsional buckling (LTB) strength of steel corrugated web I-girders (CWGs) for highway bridges.

First, the resistance of CWGs under uniform torsion is investigated. A CWG is stiffer in torsion than a conventional flat web I-girder (FWG) as a result of a “corrugation torsion” resistance. Finite element (FE) analysis shows that corrugation torsion has a complex stress distribution. A corrugation torsion model is developed based on FE analysis results. The model accurately predicts the corrugation torsion stiffness and the related flange bending moment about its weak axis.

Next, the LTB strength of CWGs under uniform bending is investigated. The elastic LTB strength of CWGs is determined by linear elastic buckling analysis. Comprehensive nonlinear FE models are developed to determine the inelastic LTB strength of CWGs, considering the effects of lateral unbraced length, initial geometric imperfections, steel stress-strain behavior, and residual stresses. The LTB strengths of selected practical CWG cases are investigated.

Then, CWG flange lateral bending induced by vertical load is investigated. Previous work shows primary shear induces flange lateral bending. The present study shows that primary bending moment also causes flange lateral bending. Based on FE analysis, a simple model is proposed to determine the maximum flange lateral bending moment due to primary bending. A design formula for flange lateral bending moment due to both primary shear and primary moment is presented.

Finally, the LTB strength of CWGs under moment gradient bending is investigated. The interaction of compression flange lateral displacement due to vertical load and due to initial geometric imperfection is studied. FE models are developed based on a practical moment gradient loading condition. The effects of vertical load induced flange lateral bending on the LTB strength are considered. The LTB strength of a CWG and a FWG under both uniform and moment gradient bending are compared.

Based on this research, design formulas for the LTB strength of CWGs are proposed and compared with AASHTO LRFD Bridge Design specifications (2004).
1 Introduction

1.1 Overview

This report addresses the lateral torsional buckling (LTB) of steel corrugated web I-girders for highway bridges. As described below, previous work has suggested that the LTB behavior of a corrugated web I-girder (CWG) is different from that of a conventional I-girder. This report presents research on the LTB of CWGs under both uniform bending and moment gradient bending. The research was based on finite element (FE) analysis of a number of CWGs with dimensions suitable for highway bridges. Design formulas for the LTB strength of CWGs were developed and are presented.

1.2 Lateral Torsional Buckling of Beams

When a perfectly straight elastic beam is bent about its strong axis, it may buckle out-of-plane by deflecting laterally and twisting at a critical value of the moment, as illustrated in Figure 1.1. This behavior is known as elastic lateral torsional buckling (LTB). The critical moment at which LTB occurs can be much lower than the plastic moment or yield moment of the beam cross section.

For a simply supported doubly symmetric I-beam under uniform bending moment, the well known theoretical elastic LTB strength (Timoshenko and Gere 1963) is

\[ M_{cr-e} = \frac{\pi}{L_b} \sqrt{\frac{EI_z}{GJ + \frac{\pi^2}{L_b^2} EI_w}} \]  

(1.1)

where \( L_b \) is the lateral unbraced length, \( EI_z \) is the bending stiffness about the weak axis, \( GJ \) is the St. Venant torsion stiffness and \( EI_w \) is the warping torsion stiffness. Tests and related research, however, showed that the LTB strength of realistic steel I-beams was less than the theoretical elastic LTB strength, because of the effects of initial geometric imperfections, yielding, and residual stresses (Trahair 1996).

Trahair and Bradford (1998) made a comparison between the LTB behavior of a real beam and various theoretical predictions, as shown in Figure 1.2, where the primary moment is plotted against the lateral deflection and twist. \( M_{cr-e} \) is the elastic LTB strength of a perfect beam. According to classical buckling theory, once this critical moment is reached, the lateral deflection and twist will increase without an increase in the primary bending moment. For an elastic beam with a small initial lateral curvature and twist, lateral deflection and twist begin immediately under loading and increase rapidly as the applied moment approaches \( M_{cr-e} \). When yielding is considered, the behavior of the beam is shown by curve A which is elastic until \( M_L \), which is the moment at which a beam without residual stresses first yields. Curve B shows the behavior of a beam without initial curvature and twist, but with residual stresses, where \( M_L \) is the inelastic LTB buckling moment. The behavior of a real
beam is shown by curve C which shows a transition from the elastic behavior of a beam with initial curvature and twist to the post-buckling behavior of a beam with residual stresses after the elastic limit is reached. $M_u$ is the ultimate moment of a real beam and serves as the basis for design equations.

1.3 Corrugated Web I-Girders

LTB is a limit state that should be considered in the design of steel corrugated web I-girders for highway bridges. A typical steel corrugated web I-girder (CWG) is composed of a steel plate top flange, a steel plate bottom flange, and a corrugated steel plate web. CWGs previously used in highway bridges have a trapezoidally corrugated web, which is illustrated in Figure 1.3. A corrugated web does not carry any significant normal stress, that is, in-plane axial stress, from primary bending moment (Elgaaly, et al. 1997, Abbas 2003) so that it does not suffer from the bend buckling. Furthermore, a corrugated web has a large shear buckling capacity (Abbas 2003) so that a thin web plate may be used without the need for transverse stiffeners, resulting in lighter weight and potentially more economical girders.

CWGs were first used in buildings in the mid 1960s. In bridges, corrugated webs have been used mainly in pre-stressed composite box girders which use a steel web with pre-stressed concrete flanges. In 1989, the Asterix Bridge was built in France. This bridge has two all steel CWGs supporting a transversely pre-stressed concrete deck (Figure 1.4). This bridge has no intermediate diaphragm or lateral bracing. In Norway, the Tronko Bridge was proposed (Figure 1.4). This bridge would have two steel corrugated webs, variable in depth, welded to steel bottom flanges. In the USA, the Pennsylvania Department of Transportations has designed and built a demonstration bridge using HPS 70W steel CWGs with a web height to thickness ratio of 250. The bridge has two continuous spans with four CWGs. The total length is 77 m (253 ft). Figure 1.5 shows the bridge during construction.

During the last fifteen years, researchers around the world have shown great interest in CWGs. The shear strength, flexural strength, fatigue strength, and fabrications of CWGs have been studied. Research shows that the flexural strength of CWGs can be controlled by one of the following limit states: flange yielding, flange local buckling (FLB), flange vertical buckling, and lateral torsional buckling (LTB).

1.4 Lateral Torsional Buckling of Corrugated Web I-Girders

For a conventional steel I-girder, it is well known that the strength under primary bending can be reduced to significantly less than the plastic moment due to LTB. There are two distinctive aspects of CWG behavior that make the LTB behavior of a CWG different from that of a conventional I-girder. On one hand, vertical loading on a CWG induces both in-plane and out-of-plane bending and twisting of the flanges (Lindner and Aschinger 1990, Lindner 1992, Aschinger and Lindner 1997 and Abbas 2003). As a result, the potential for LTB may be increased for a CWG compared to a conventional I-girder, and the LTB strength of a CWG may be smaller than that of a conventional I-girder. On the other hand, Lindner and Aschinger (1990) showed that a CWG has an increased torsional stiffness compared to a conventional I-girder. The
increased torsional stiffness decreases the potential for LTB, and the LTB strength of a CWG may be larger than that of a conventional I-girder. These two aspects of CWG behavior leads to different possible results regarding LTB strength.

Lindner and Aschinger (1990) studied the torsional stiffness of CWGs, assuming that the St. Venant torsion stiffness does not change. To capture the increased torsional stiffness, they defined the warping torsion stiffness as

\[ I'_w = I_w + c_w \frac{L^2}{E \pi^2} \]  (1.2)

where \( I_w \) is the warping torsion constant, calculated using the formula for a conventional I-girder, \( L \) is the span length and \( c_w \) is defined as

\[ c_w = \frac{h^2 h_r^2}{8 u_x (b + d)} \]

where \( h \) is the distance between the flange centroids, \( h_r \) is the corrugation depth, \( b \) is the length of the longitudinal fold, \( d \) is the projection of the inclined fold on the longitudinal axis (see Figure 1.3) and

\[ u_x = \frac{h}{2 G b t_w} + \frac{h^2 (b + d)^3}{600 E b^2} \left( \frac{1}{I_{yf}} + \frac{1}{I_{ybf}} \right) \]

where \( t_w \) is the web thickness and \( I_{yf} \) and \( I_{ybf} \) are the top and bottom flange moments of inertia about the weak axis of the flanges. Equation (1.2) was derived for a simply supported span loaded with a concentrated torque at the mid span. The twist angle shape is assumed to be half sine wave along the girder.

Lindner and Aschinger proposed several approaches to determine the LTB strength of CWGs (Lindner 1990, Lindner and Aschinger 1990, Lindner 1992, and Aschinger and Lindner 1997). The first approach (termed Method A) is summarized here. Method A uses the design equations for a conventional I-girder with the modified warping torsion constant, \( I'_w \). The nominal LTB strength is calculated by the rules of DIN 18800 part 2 as

\[ M_n = \kappa M_p \]  (1.3)

where \( M_p \) is the plastic moment of the CWG, calculated without any contribution from the web, and \( \kappa \) is a reduction factor defined as

\[ \kappa = \left( \frac{1}{1 + \lambda^2 M} \right)^{\frac{1}{n}} \]  (1.4)

where \( n \) is 2.0 for a welded beam. \( \lambda_M \) is a slenderness ratio defined as

\[ \lambda_M = \sqrt{\frac{M_p}{M_{cr,e}}} \]  (1.5)

where \( M_{cr,e} \) is the theoretical elastic LTB strength calculated using the modified warping torsion constant, \( I'_w \).
Eight tests on CWGs were reported by Aschinger and Lindner (1997) and the test results are compared to the results from Method A in Table 1.1, where $M_{exp}$ is the LTB strength from the test expressed as the peak primary bending moment. The test girders were simply supported with a concentrated load applied on the top flange at the mid span. The test setup allowed the load to be applied vertically when the girder deforms laterally and torsionally (Lindner and Aschinger 1990). $M_n$ is the strength calculated from Equation (1.3). In their calculations, two values for the parameter $n$ were considered: $n = 1.5$ and $n = 2.0$. Table 1.1 shows that the calculated results for test Kp6/1 and Kp7/1 overestimated the test results by as much as 27% and Aschinger and Lindner (1997) recommended that the LTB capacity from Method A should be reduced by a factor of 0.8.

Ibrahim (2001) proposed to calculate the LTB strength of CWGs using the LTB strength design formulas for conventional I-girders from the AISC LRFD Specifications (1994) with the modified warping torsion constant, $I'_w$, shown in Equation (1.2). Two simply supported CWGs were tested with a concentrated load applied on the top flange at the mid span. Lateral support of the top flange was provided by friction between the jack head and the flange. The results from the tests, FE analyses, Lindner’s Method A, and the AISC LRFD Specification design formulas for the two CWGs are compared in Figure 1.6. It can be seen that the result from the AISC formulas is in good agreement with both the test and FE analysis results for specimen LTB5C11. The AISC formulas overestimate the LTB strength of specimen LTB8C11 compared to the test and the FE analysis results.

The above discussion shows that the previously developed approaches to calculate the LTB strength of CWGs need further improvement. In addition, the past research has been conducted on CWGs with dimensions that do not fall within the typical range for steel I-girders for bridges. It is therefore necessary to conduct a comprehensive study of the LTB strength of CWGs considering practical dimensions of steel highway bridge girders.

### 1.5 Research Objectives

The research described in this report focuses on the study of the LTB strength of steel CWGs, with emphasis on their use as highway bridge girders. The research objectives are:

- To investigate the uniform torsion of CWGs and to develop an analytical model which is capable of predicting the torsional stiffness of CWGs.
- To investigate the LTB behavior of CWGs under uniform bending moment.
- To investigate flange lateral bending of CWGs under vertical load.
- To investigate the LTB behavior of CWGs under moment gradient bending.
- To develop design formulas for the nominal LTB strength of CWGs, which can be used to design CWGs for highway bridges.
1.6 Research Scope

To accomplish these objectives, detailed FE models of CWGs were developed. These models consider all the major factors that determine the LTB strength of CWGs. These factors include the lateral unbraced length, initial geometric imperfections, steel stress-strain behavior, and residual stresses. Detailed studies of these factors were made. In addition, to enable the torsional stiffness of CWGs to be easily and accurately estimated, a detailed study of the behavior of CWGs under uniform torsion was made. The FE models were developed and analyzed using the general purpose FE analysis package ABAQUS versions 6.3 and 6.5. The CWGs considered in these studies have identical flanges with a trapezoidally corrugated web.

1.7 Critical Assumption

A critical assumption that is used repeatedly in this research is that the corrugated web does not carry any normal force or normal stress along the girder longitudinal axis. In particular, the longitudinal web plates, parallel to the girder longitudinal axis, and the inclined web plates, are assumed to not carry axial normal stress due to overall girder bending or axial force, or due to local stresses that develop under torsion. Normal force or normal stress in the vertical direction of the web is expected. Based on this assumption, the primary bending moment on the CWG is carried entirely by the flanges with no contribution from the web. This assumption has been investigated in detail by Abbas (2003), and was validated.

1.8 Report Outline

The report is written in six chapters:

1. Introduction.
2. Uniform Torsion of Corrugated Web Girders.
3. Lateral Torsional Buckling Under Uniform Bending.
4. Flange Lateral Bending under Vertical Load.
5. Lateral Torsional Buckling Under Moment Gradient Bending.

The system of units used in the report is the SI system with the basic units of kilo-Newton (kN) for force, millimeter (mm) for length, and mega-Pascal (MPa) for stress. The remaining chapters are organized as follows:

Chapter 2 presents the uniform torsion of CWGs. First, the uniform torsion of a prototype CWG and a comparable conventional I-girder were studied using FE analyses. By studying the deformation and strain energy, the behavior leading to the increased torsional stiffness of CWGs is identified and this behavior is defined as “corrugation torsion”. The complicated nature of the behavior of CWG under uniform torsion is revealed by the internal force patterns. Static equilibrium formulations show that corrugation torsion is highly statically indeterminate. A corrugation torsion model is proposed based on the FE analysis results. A static solution is developed to determine the corrugation torsion stiffness. Regression analyses of FE analysis results are used to provide parameters for the corrugation torsion model.
Chapter 3 presents the LTB of CWGs under uniform bending. Linear elastic buckling analysis results for the elastic LTB strength of CWGs are presented. To determine the inelastic LTB strength, comprehensive finite element models were developed which consider the effects of lateral unbraced length, initial geometric imperfections, steel stress-strain behavior, and residual stresses. The effects of these parameters are demonstrated. The LTB strengths of eight different CWG cases were determined from FE analyses for a broad range of lateral unbraced lengths, and from these results, design formulas to calculate the LTB strength of CWGs were developed and are presented.

Chapter 4 presents a study of flange lateral bending in CWGs due to the primary shear and the primary bending moment produced by vertical load. First, the flange lateral bending induced by primary shear in the presence of the intermediate lateral braces is presented using the fictitious load approach developed by Abbas (2003). Second, based on the results of FE analysis, a simple model is proposed to determine the flange lateral bending induced by primary bending. The results are combined with the results for flange lateral bending due to primary shear to provide the total flange lateral bending under vertical load, which can be used in design.

Chapter 5 presents the LTB of CWGs under moment gradient bending. The interaction of the compression flange lateral displacement induced by the vertical load with the flange lateral displacement from initial geometric imperfection is presented first. FE models for LTB of CWGs are presented which consider practical moment gradient loading conditions. The LTB strength of CWGs was studied in detail for two moment gradient conditions considering the effects of vertical load induced flange lateral bending moment and the results are presented. Design formulas are developed and presented, and these formulas are compared with similar formulas from the AASHTO LRFD Bridge Design Specifications (2004).

Finally, Chapter 6 summarizes this research, and presents conclusions, findings, contributions from the research, as well as recommendations for future work.
Table 1.1 Comparison of test results and LTB strength equations
(from Aschinger and Lindner 1997)

<table>
<thead>
<tr>
<th>Test Results</th>
<th>$M_{exp}/M_n$</th>
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<tbody>
<tr>
<td></td>
<td>($n = 1.5$)</td>
</tr>
<tr>
<td>Kp 5/1</td>
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</tr>
<tr>
<td>Ks 5/1</td>
<td>1.05</td>
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<tr>
<td>Kp 6/1</td>
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<td>Mean</td>
<td>1.09</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Figure 1.1 Lateral torsional buckling of I-beams (from Timoshenko 1963)

Figure 1.2 Behavior of ideal and real beams (from Trahair and Bradford 1998)
Figure 1.3 Corrugated web I-girder with trapezoidal web (from Abbas et al. 2002)

Figure 1.4 Examples of corrugated web girder bridge
(from Cheyrezy and Combault 1990)
Figure 1.5 PennDOT CWG demonstration bridge during construction
Figure 1.6 Comparison of results from tests, FE analysis, Lindner’s Method A, the AISC formulas (reproduced from Ibrahim 2001)

(a) Specimen LTB5C11

(b) Specimen LTB8C11
2 Uniform Torsion of Corrugated Web Girders

The torsional resistance of conventional flat web I-girders includes St. Venant torsion (or uniform torsion) resistance and warping torsion (or non-uniform torsion) resistance. Lindner and Aschinger (1990) showed that for corrugated web girders the torsional resistance is increased compared to that of conventional I-girders, because interaction between the web folds and the flanges cause out-of-plane bending of the flanges. Lindner and Aschinger (1990) quantified this increased torsional resistance as an increased warping torsion stiffness.

Analyses presented in this chapter show that this unique feature of corrugated web girder torsion is best quantified as an increase in St. Venant torsion stiffness. In this chapter, the uniform torsion of corrugated web girders is studied and a uniform torsion model based on FE analysis is proposed.

2.1 Finite Element Analysis

2.1.1 Finite Element Model

As noted in Chapter 1, the scope of this report is limited to steel corrugated web I-girders with a trapezoidally corrugated web (see Figure 1.3), which consists of longitudinal folds (parallel to the longitudinal axis of the girder) and inclined folds (inclined to the longitudinal axis). Since the uniform torsion of conventional I-girders is well established, it is useful to compare the torsion of a corrugated web girder (denoted CWG) with that of a comparable conventional flat web I-girder (denoted FWG), which has the same flange width and thickness and the same web depth and thickness.

Finite element (FE) simulations are used for a study of CWG behavior under uniform torsion. The commercial, general-purpose FE package ABAQUS v6.3 is used. The study initially focuses on a CWG designed using the AASHTO LRFD Bridge Design Specifications (2000) and CWG design criteria developed at the ATLSS Center at Lehigh University (Sause et al. 2003). The girder, denoted the prototype corrugated web girder (prototype CWG) was designed for a simply supported bridge with a 40 m span and 15 m wide concrete deck as shown in Figure 2.1. The bridge has four straight CWGs spaced at 3.8 m.

The prototype CWG cross section and the corrugation dimensions are shown in Figure 2.2. The length of the longitudinal fold $b$ is 450 mm, the length of the inclined fold $c$ is 424 mm and the corrugation angle $\alpha$ is 45 degree. From these three basic parameters, the other corrugation parameters can be derived, which include the corrugation depth $h_r$ (300 mm) and the projected axial length of one corrugation $L_e$ (1500 mm). The flange is 500 by 50 mm and the web is 1550 by 10 mm, with a $D/t_w$ ratio of 155, which is a relatively stocky corrugated web, since typical $D/t_w$ ratios for corrugated webs range between 200 and 500.

An FE model of the prototype CWG including 10 corrugations (15m in length) was created and used to study CWG uniform torsion. The left end of the model is fixed against twist and a twist angle of 0.05 radian is applied at the right end to create
a uniform torsion loading condition. By trial and error, element S8R, which is an eight-node thick shell element with 4 integration points, was selected to model both the flange and web. Since it is a thick shell element, the through-thickness shear force (stress) can be obtained from this element.

The two flanges and the web are connected using the surface based multi-point constraint (MPC) option in ABAQUS Version 6.3. Element based surfaces are defined for the flanges and node based surfaces are defined for the web nodes which are connected to the flanges. The flanges and web are then connected by joining the two surfaces together. By using this constraint option, the flanges and web are not connected node-by-node, which enables the FE meshes to be changed conveniently.

The FE model of the prototype CWG is supported at the node at the center of web at both ends, which is called the support node. The displacements of the other nodes on the end cross section are constrained to the displacements of the support node using the “kinematic coupling” option of ABAQUS Version 6.3. Figure 2.3 shows both the global axes and local axes used in the model and defines the degrees of freedom (DOF). Note that the degrees of freedom are defined with respect to the global axes. DOF 1 to 4 of the left support node are restrained and DOF 2 to 4 of the other nodes on the left cross section are kinematically coupled to this reference node. The same boundary conditions are used for the right end except that only DOF 2 and 3 of the right support node are restrained. Therefore, the left end is restrained from twist and the right end is free to twist about the longitudinal axis. These boundary conditions also leave the flanges free to warp. For the comparable FWG, these boundary conditions produce torsional shear stresses similar to those from St. Venant torsion theory.

An optimized FE mesh for the prototype CWG model was developed by comparing the results of FE models with different meshes (a so-called mesh convergence test) as shown in Table 2.1. The optimized mesh produces converged results but has the least number of elements. Table 2.1 lists the number of elements per flange within the length of one corrugation in the local axis 1 and axis 2 directions, the number of elements per longitudinal fold in the local axis 1 and axis 2 directions, and the number of elements per inclined fold in the local axis 1 and axis 2 directions, where the local axes are defined in Figure 2.3. For example, for Model 12, the FE mesh of the bottom flange and the web within the length of one corrugation is shown in Figure 2.4(a). Within the length of one corrugation, the flange has 12 elements in the local axis 1 direction and 4 elements in the local axis 2 direction. Both the longitudinal fold and the inclined fold have 4 elements in their local axis 1 direction and 16 elements in their local axis 2 direction. Table 2.1 also lists the total number of elements within the length of one corrugation and the reaction torque for each model that was studied.

For Model 11 to Model 54 (except Model 33) a uniform mesh is used for both the flanges and the web. For the FE mesh of Model 12 shown in Figure 2.4(a), the mesh is uniform for the flanges, for every longitudinal fold and inclined fold. For Model 33, the mesh is uniform for the flanges and is non-uniform for the web, where the height of the elements at locations close to flanges is one quarter of the height of elements at locations away from the flanges, as shown in Figure 2.4(b). For Model 61
to Model 94, a uniform mesh is used for the flanges and a biased mesh is used for the web by specifying a bias factor, and the mesh for Model 63 is shown in Figure 2.4(c). The bias factor, $b_{bias}$, is the ratio of adjacent distances between nodes along each line of nodes generated as the nodes go from the first bounding node set to the second. The value of $b_{bias}$ must be positive. The bias intervals along the line from the first bounding node, which is connected to the flanges, toward the web mid-depth, are $L_1$, $L_1/b_{bias}$, $L_1/b_{bias}^2$, $L_1/b_{bias}^3$, … , where $L_1$ is the length of the first interval. By specifying a bias factor smaller than 1, the web nodes are concentrated toward the flanges so that the closer to the flanges, the smaller the element is.

From Model 11 to Model 16, it can be seen that the reaction torque decreases when both the flange and web meshes become denser and converges at the value of 18160 kN-mm. From Model 21 to 24, the web mesh is kept constant while the flange mesh is changed. It can be seen that doubling the number of elements in the longitudinal direction is more effective than doubling the number of elements in the transverse direction. Model 32 and Model 33 have the same flange mesh but a different web mesh. As mentioned above, the web mesh of Model 33 is non-uniform. It can be seen that the non-uniform web mesh is very effective and significantly reduces the total number of elements. Model 51 to Model 54 and Model 61 to Model 64 have the same flange mesh and the numbers of web elements are the same. Model 51 to Model 54 have a uniform web mesh while Model 61 to Model 64 have a biased web mesh with a bias factor of 0.8. It can be seen that the reaction torque of Model 61 to Model 64 converges faster. Results of Model 71 to Model 73 show that even when a biased web mesh is used, the flange mesh is still needs to be fine enough for the reaction torque to converge. Since an 8-node element is used, there are two intervals on every side of the element. For Model 61 to Model 64, the two intervals are not equal with the smaller interval located closer to the nearby flange, as shown in Figure 2.5(a). The two intervals in the vertical direction can be made equal (Model 91 to Model 94), as shown in Figure 2.5(b). Comparing the reaction torque, it can be seen that Model 61 to Model 64 are more efficient.

Due to the complicated stress transfer between the flanges and the web, a finer mesh is needed for the flanges and the part of the web located close to the flanges. The biased web mesh serves this purpose well and is very effective at reducing the total number of elements. It takes 1624 elements within the length of one corrugation for a model with the uniform mesh (Model 14) but only 576 elements for a model with the biased web mesh (Model 62) to produce the converged reaction torque.

From above analysis, Model 63 is selected as the optimized mesh for the CWG uniform torsion study, as shown in Figure 2.4(c). There are 24 elements in the local axis 1 direction and 8 elements in the local axis 2 direction within the length of one corrugation for each flange. On every longitudinal fold and inclined fold, there are 6 elements in the local axis 1 direction and 12 elements in the local axis 2 direction. The size of the web element in the local axis 2 direction is biased with a bias factor of 0.8. The total number of elements within the length of one corrugation is 672.

For the FE model of the comparable FWG, the mesh selected for the prototype CWG is used for flanges. The element size in the vertical direction is also the same,
but a uniform mesh is used in the longitudinal axis direction. As a result, the nodes on the flanges and the web are aligned with each other on the flange-web interfaces. The boundary conditions, constraints and loading conditions are the same as those of the FE model for the prototype CWG. The total number of elements in the two models is also the same.

2.1.2 Comparison of the Prototype CWG and the FWG under Uniform Torsion

The torsional deformations of the prototype CWG and the FWG are shown in Figure 2.6. The Mises stresses on the lower surface of the shell elements, e.g. the bottom of the top flange and the bottom of the bottom flange, are shown in Figure 2.7. The Mises stress is a notional stress defined as

$$Mises = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22} + 3\sigma_{12}^2}$$

for a plane stress state. For the FWG, the applied twist produces a torsional deformation that is uniform along the length and the stresses are as St. Venant torsion theory predicts. For the CWG, the overall torsional deformation is similar to that of the FWG but the stresses are quite different. It can be seen from Figure 2.7 that the stresses are similar from one corrugation to the next. The reaction torque for the CWG, equal to 18160 kN-mm, is 1.79 times larger than that of the FWG, equal to 10140 kN-mm. The torsional stiffness of the CWG is increased significantly due to the corrugated web.

Figure 2.8 shows an end view of the torsional deformation of the FWG and the CWG. Again, the overall torsional deformation of the two girders is similar. But it can be seen that there are wavelike flange bending deformations in the CWG. To further study these deformations, the vertical deflection of the bottom flange along three axes of the bottom flanges of the girders are compared. The locations of these axes (U, M and L) are shown in Figure 2.9. Figure 2.10 shows that the vertical deflections of the CWG bottom flange along these three axes are equivalent to those of the FWG plus wavelike deflections. The vertical deflection differences along these three axes are shown in Figure 2.11. It can be seen that the deflection differences are similar along these three axes. Figure 2.11 shows that the flange of the CWG is pushed up and pulled down at locations which are centered on the centers of the inclined folds. The magnitude of the flange out-of-plane deformation is almost constant along the girder. The same behavior is observed for the top flange.

Compared to the FWG, an additional torque is needed to provide the work needed to bend flanges out-of-plane and this produces the increased torsional stiffness of the CWG.

2.1.3 Strain Energy Study

In this section, an analysis of the strain energy in the CWG under uniform torsion is conducted. A study of the CWG stresses induced by uniform torsion was made and it was observed that the stresses are essentially the same from one corrugation to another. Therefore, the strain energy of a single corrugation, the fifth corrugation, which is next to the mid span, was studied. Table 2.2 lists the strain
energy from the flanges, the inclined web folds and the longitudinal web folds for the
FE model of the prototype CWG described in the previous section, which has 50mm
thick flanges. As discussed later, other flange thicknesses were also studied.

The strain energy was calculated as follows. The FE analysis results show that
the strain energy associated with plate through thickness shear stresses is small and it
was not included in the calculations. The strain energy in a plate due to in-plane
stresses is defined as (Ugural and Fenster 1995)

\[
U = \int \frac{1}{2E} \left( \sigma_{11}^2 + \sigma_{22}^2 - 2\nu \sigma_{11} \sigma_{22} \right) dV + \frac{1}{2G} \sigma_{12}^2 \tag{2.1}
\]

where \(\sigma_{11}\) is the normal stress in local axis 1 direction, \(\sigma_{22}\) is the normal stress in
local axis 2 direction and \(\sigma_{12}\) is the in-plane shear stress. From thick shell (Mindlin)
theory (Cook 1995), \(\sigma_{11}\), \(\sigma_{22}\) and \(\sigma_{12}\) are linear over plate thickness so that they can
be decomposed as

\[
\begin{align*}
\sigma_{11} &= \sigma_{11P} + \sigma_{11B} \\
\sigma_{22} &= \sigma_{22P} + \sigma_{22B} \\
\sigma_{12} &= \sigma_{12T} + \sigma_{12T}
\end{align*} \tag{2.2}
\]

where \(\sigma_{11P}\), \(\sigma_{22P}\) and \(\sigma_{12T}\) are the average stresses over the plate thickness, \(\sigma_{11B}\) and
\(\sigma_{22B}\) are the plate bending stresses that vary linearly over the plate thickness and are
zero at the plate mid-surface, and \(\sigma_{12T}\) are St. Venant torsion shear stresses, that also
vary linearly over the plate thickness and are zero at the plate middle surface. The
strain energy \(U\) can be expressed then as

\[
U = U_b + U_p + U_{1T} + U_{2T}
\]

with the strain energy terms, \(U_b\), \(U_p\), \(U_{1T}\), and \(U_{2T}\) defined as follows. \(U_b\) is the
strain energy associated with plate bending and is defined as

\[
U_b = \int \frac{1}{2E} \left( \sigma_{11B}^2 + \sigma_{22B}^2 - 2\nu \sigma_{11B} \sigma_{22B} \right) dV \tag{2.3}
\]

where \(\nu\) is the Poisson’s ratio. In terms of plate bending curvature, \(U_b\) is defined as

\[
U_b = \frac{D}{2} \int \left[ \left( \frac{\partial^2 w}{\partial l^2} \right)^2 + \left( \frac{\partial^2 w}{\partial 2^2} \right)^2 + 2\nu \frac{\partial^2 w}{\partial l^2} \frac{\partial^2 w}{\partial 2^2} \right] dld2
\]

where \(\frac{\partial^2 w}{\partial l^2}\) and \(\frac{\partial^2 w}{\partial 2^2}\) are the plate bending curvature about local axis 2 and
local axis 1 and \(D\) is the plate flexural rigidity.

\(U_p\) is the strain energy associated with plate in-plane tension or compression
and is defined as

\[
U_p = \int \frac{1}{2E} \left( \sigma_{11P}^2 + \sigma_{22P}^2 - 2\nu \sigma_{11P} \sigma_{22P} \right) dV
\]

\(U_{1T}\) is the strain energy associated with St. Venant torsion shear stresses \(\sigma_{12T}\)
and is defined as
$U_{i1} = \int \frac{1}{2G} \sigma_{i1}^2 dV$

$U_{i2}$ is the strain energy associated with the average plate in-plane shear $\sigma_{i2}$ and is defined as

$U_{i2} = \int \frac{1}{2G} \sigma_{i2}^2 dV$

Rather than work directly with the stresses from the FE analyses, the section forces, moments and curvatures were used to calculate the strain energy. The section forces and moments are obtained by integrating the stresses through the thickness. The section forces and moments are defined in ABAQUS v6.3 as

$\left( SF1, SF2, SF3, SF4, SF5 \right) = \int_{-t/2}^{t/2} \left( \sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{13}, \sigma_{23} \right) dz$ \hspace{1cm} (2.4)

$\left( SM1, SM2, SM3 \right) = \int_{-t/2}^{t/2} \left( \sigma_{11}, \sigma_{22}, \sigma_{12} \right) zdz$

where $t$ is the plate thickness. $SF1$ and $SF2$ are the normal forces per unit length in local axis 1 and axis 2 direction, respectively. $SF3$ is the in-plane shear force per unit length, and $SF4$ and $SF5$ are the through-thickness shear force per unit length. $SM1$ and $SM2$ are the plate bending moments per unit length about local axis 2 and axis 1, respectively. $SM3$ is the torque per unit length due to the in-plane shear stress. Each section force has the unit of kN/mm and each section moment has the unit of kN.

The section curvatures or twists are also obtained from the FE analysis, and are defined in ABAQUS v6.3 as

$SK1 = \frac{\partial^2 w}{\partial l^2}$

$SK2 = \frac{\partial^2 w}{\partial 2^2}$

$SK3 = 2 \frac{\partial^2 w}{\partial l \partial 2}$

where 1 and 2 are the two in-plane local axes as shown in Figure 2.3.

The section forces, moments, and curvatures were obtained at the four integration points of the S8R element used in the FE model. To calculate the strain energy simply, the section forces, moments, and curvatures were assumed to be constant over one quarter of the element and equal to the values at the integration point within that quarter of the element, and the total strain energy of the element is the sum of the strain energy of the four quarters. The strain energy for the length of one corrugation of the CWG is the sum of the strain energy of all elements within the one corrugation length. Using the section forces, moments, and curvatures at the integration points and using the simple method of summing the strain energy, the strain energy terms were calculated as follows.
\[ U_b = \sum_{i=1}^{n} \sum_{j=1}^{4} \frac{D_i A_i}{8} \left( SK_{1,i,j}^2 + SK_{2,i,j}^2 + 2\nu SK_{1,i,j} SK_{2,i,j} \right) \]
\[ U_p = \sum_{i=1}^{n} \sum_{j=1}^{4} \frac{A_i}{8Et_i} \left( SF_{1,i,j}^2 + SF_{2,i,j}^2 - 2\nu SF_{1,i,j} SF_{2,i,j} \right) \]
\[ U_{n1} = \sum_{i=1}^{n} \sum_{j=1}^{4} \frac{A_i}{16} (1-\nu) SK_{3,i,j}^2 \]
\[ U_{n2} = \sum_{i=1}^{n} \sum_{j=1}^{4} \frac{A_i}{8Et_i} SF_{3,i,j}^2 \]

where \( n \) is the total number of elements within the length of one corrugation, \( D_i \) is the plate flexural rigidity of element \( i \), \( A_i \) is the area of element \( i \), \( SK_{i,j} \) and \( SF_{i,j} \) are the section curvature and section force at integration point \( j \) of element \( i \), respectively.

As noted earlier, the strain energy associated with plate through thickness shear stresses \( \sigma_{13} \) and \( \sigma_{23} \) was not calculated for the FE model of the prototype CWG, the external work done by the reaction torque acting through the imposed twist, \( W \), equal to 45.4 kN-mm, was sufficiently close (within 2%) to the total strain energy \( U \), equal to 44.5 kN-mm, for the simple method of calculating the strain energy to be considered acceptable, and for the strain energy related to \( \sigma_{13} \) and \( \sigma_{23} \) to be considered negligible.

Table 2.2 shows that the two most important strain energy components for the prototype CWG are \( U_b \) and \( U_{n1} \), which account for 95.5% of the total strain energy. \( U_b \) is due almost entirely to flange plate bending alone (18.12 out of 18.13 kN-mm). The strain energy due to the flange plate bending about its transverse axis (i.e., from \( \sigma_{11b} \)) is 18.8 kN-mm, which is close to the total strain energy due to flange plate bending (18.12 kN-mm). Strain energy due to \( \sigma_{11b} \) is larger that the total because the contribution of the second and third terms in Equation (2.3) is negative. This comparison shows that the strain energy due to flange plate bending about its transverse axis is the main component of \( U_b \). Comparing \( U_{n1} \) for the prototype CWG (24.4 kN-mm) to the external work \( W \) for the FWG, equal to 25.4 kN-mm, it can be seen that \( U_{n1} \) for the prototype CWG is similar to the strain energy, \( U \), of the FWG under uniform torsion.

To study the effects of flange thickness, the flange thickness of the prototype CWG was varied to 40mm and to 20mm. FE models of CWGs with the 40mm and 20mm thick flanges were developed. Similar strain energy studies were done for these two FE models and the calculated strain energy is shown in Table 2.3 and Table 2.4, respectively. It can be noted that the conclusions for the FE model of the prototype CWG also applies to the FE model of the CWGs with 40mm and 20mm thick flanges.

The strain energy analyses show that the torsional resistance of a CWG under uniform torsion has two main components. The first is similar to the torsional
resistance of a FWG under uniform torsion and is called the “St. Venant torsion” resistance in this report. The second is the torsional resistance that is unique to a CWG, which is related to the flange out-of-plane bending, and is called “corrugation torsion” resistance in this report.

2.1.4 Kinematics and Statics of Corrugation Torsion

The FE analysis results show that the torsional stiffness of a CWG is increased relative to that of a FWG by the corrugation torsion resistance. In this section, the corrugation torsion resistance is related to the flange out-of-plane bending by simple kinematic and static analyses.

Figure 2.12 shows the top view and side view of the torsional deformation of a single corrugation. The top view shows that the top and bottom flange rotate to opposite directions. The centerline MM of each flange does not move in the longitudinal direction of the girder. Line 11 moves longitudinally to the right and line 22 moves to the left. On the bottom flange, at the corresponding locations, the movement of similar lines is in the opposite direction. For the longitudinal fold B (L.F.B) shown in Figure 2.12(a), the top edge moves along with the top flange to the right and the bottom edge moves to the left. As a result, longitudinal fold B has a clockwise rotation, as shown in Figure 2.12(b). On the contrary, longitudinal fold A (L.F.A) has a counter-clockwise rotation. To maintain deformation compatibility, flanges over inclined fold 1 (I.F.1) are bent upwards and flanges over inclined fold 2 (I.F.2) are bent downwards. This kinematic analysis shows how uniform torsion deformation of the flanges is related to the observed flange out-of-plane bending, and is called the kinematics of corrugation torsion.

Corrugation torsion can also be understood qualitatively from a simple static model developed by Lindner and Aschinger (1990) for a simply supported CWG loaded with a concentrated torque at the mid span. This model is more appropriate for the uniform torsion of CWGs since all the internal forces used in this model are associated with corrugation torsion. Figure 2.13 shows stress resultants related to the corrugation torsion. Figure 2.13(a) shows the shear stress resultants at the web-flange interface acting on the flanges. On the cross section at the center of an inclined fold, the torque from the corrugation torsion resistance is related to the flange transverse shear $V_{fy}$ by

$$T_c = V_{fy} h$$

where $h$ is the distance between the flange centroids. From the flange in-plane moment equilibrium

$$V_{fy} L_c = V_{wl} h_r$$

where $L_c$ is the projected axial length of one corrugation and $V_{wl}$ is the shear between the longitudinal web fold and the flange. Figure 2.13(b) shows the shear forces acting at the foldline between the longitudinal fold and the inclined fold. From the in-plane moment equilibrium of longitudinal fold B (L.F.B),

$$V_{wz} b = V_{wl} h$$
where $b$ is the length of the longitudinal fold and $V_{wz}$ is the web vertical shear between the longitudinal and inclined fold. Figure 2.13(b) shows that both longitudinal fold A and B induce upward shear forces on inclined fold 1 (I.F.1). To maintain the vertical force equilibrium on inclined fold 1, both the top and bottom flange induce two downward forces $P$ on it, each of which equals $V_{wz}$ in magnitude. Similarly, both the top and bottom flange induce two upward forces $P$ on inclined fold 2 to keep it in equilibrium. As a result, the flanges are subjected to these vertical forces that alternate in direction.

Lindner and Aschinger (1990) modeled each flange as a series of simply supported beams that are supported at the centers of the longitudinal folds, as illustrated in Figure 2.14. The vertical forces $P$ from the inclined folds are treated as concentrated forces acting at the mid span of the simply supported beams. The directions of the forces are alternating, so the flange is alternately pulled down and pushed up along the girder length.

This model is statically determinate, and the internal forces can be related to the torque from corrugation torsion, $T_c$. Lindner and Aschinger (1990) then determined the corrugation torsion stiffness through an energy approach by equating the external work done by the torque $T_c$ to the internal strain energy. In the model, the flange is supported only at the centers of the longitudinal folds. In reality, however, the flange is supported continuously by the corrugated web. However, this model shows qualitatively the relationship of the torque from corrugation torsion to the flange out-of-plane bending.

### 2.1.5 Detailed Analysis of Corrugation Torsion Stresses

The static model of Lindner and Aschinger (1990) does not include a shear force at the interface between the inclined fold and the flange. In addition, the variation of normal stress and shear stress along at the interface of the longitudinal fold and flange is not considered. As a result, the flange bending about its weak axis varies linearly along the girder. FE analysis results show that the internal force variation is far more complicated than assumed in the model. In this section, the internal forces within the length of one corrugation of a CWG under uniform torsion are studied, especially the internal forces related to corrugation torsion. Figure 2.15 shows the fifth corrugation of the prototype CWG FE model described earlier. Nine cross sections are identified and numbered for discussion purposes. Fold lines and center of inclined fold are also labeled.

Contour plots of the section forces at the nodes from FE analysis were made using the program Surfer 6.03. Figure 2.16 shows the section force contour plots on the top and bottom flanges, where BF stands for the bottom flange and TF stands for the top flange. Positions of critical sections and the corrugation are also shown and the critical sections are identified by text boxes. SF1, SF2, SF3, SM1, SM2, and SM3 were defined in Equation (2.4), and the local directions for the flanges are shown in Figure 2.3. Selected bottom flange section forces are also plotted on five cross sections in Figure 2.17. The positions of the five cross sections are shown in Figure 2.18(a),
where section 67M is halfway between section 6 and 7 and section 78M is halfway between section 7 and 8. Several observations are made from these figures as follows.

- Uniform torsion of CWG produces complex stress states.
- Dramatic changes in stresses occur where the web and flanges are connected, which is probably the result of concentrated stress transfer between the web and flanges.
- For SF1, SF2 and SF3, the magnitudes are the same on the two flanges while the signs are opposite. For SF4, SM1, SM2 and SM3, the magnitudes and signs are same on the two flanges.
- On either flange, the section forces are 2-fold rotational symmetric about the point O, which is identified in Figure 2.15(a). Over half of the corrugation length, the section forces are symmetric about section 6 (or 2). Due to the symmetries, when the section forces on a quarter of a corrugation (e.g. between section 6 and 8) are known, the section forces at other locations can be determined.

Other observations specific to each section force are as follows.

- The maximum values of SF1 are at the locations close to the web foldlines (section 1, 3, 5, and 7).
- The maximum values of SF3 are at the locations of inclined folds. On a typical flange cross section, SF3 is near zero at the two flange edges and increases towards flange center-line (see Figure 2.17(b)).
- The maximum values of SF4 are at the flange edges (see Figure 2.16 and Figure 2.17(c)).
- The maximum values of SM1 are at the centers of the inclined folds (sections 0, 4, and 8) and are zero at the centers of longitudinal folds (sections 2 and 6) (see Figure 2.16). The magnitude of SM1 increases from section 6 to section 8. SM1 is also fairly uniform across the flange width (see Figure 2.17(d)).
- The values of SM3 approach zero towards flange edges and are pretty uniform at other locations with the maximum values at the centers of the inclined folds. SM3 on the six cross sections are different from each other although the magnitudes are fairly close (see Figure 2.17(e)).

Figure 2.19 shows the contour plots for three web in-plane section forces, SF1, SF2, and SF3. Positions of critical sections and the web mid-depth are also shown and are identified by text boxes. The local directions for the web are shown in Figure 2.3. The horizontal axis in these plots is the projection of the web in global X direction.

Web section forces on seven cross sections are also shown in Figure 2.20. The positions of the seven cross sections are shown in Figure 2.18(b) where 7L and 7R are right next to section 7, the foldline. Several observations are made from these figures as follows.

- The web section forces are not uniform and are larger at locations close to the flanges, especially around the foldlines.
- The web section forces are symmetric about the web mid-depth and are symmetric about section 4. Over half of the corrugation length, the section forces are symmetric about section 6 (or 2). Due to the symmetries, when the
section forces on one quarter of a corrugation above or below the web mid-depth are known, the section forces at other locations can be determined. Other observations specific to each section force are as follows.

- The maximum values of SF1 are at the corners of the longitudinal folds. SF1 is either zero or very small at locations away from the flanges (see Figure 2.19). SF1 is zero at section 6 and on section 7 and section 7L at locations close to the flanges, SF1 is much higher than that at other locations. On section 78M and section 8, the sign of SF1 is reversed at locations close to the flanges (see Figure 2.20(a)).
- The maximum values of SF2 are at the centers of the inclined folds (see Figure 2.19). SF2 is very small on the longitudinal folds and is zero at section 6. At section 67M and section 7L, the sign of SF2 is reversed at locations close to the flanges (see Figure 2.20(b)).
- SF3 is zero at section 8 and is much higher on section 7 at locations close to the flanges than that at other locations. At an arbitrary cross section, SF3 at locations away from the flanges is relatively uniform. At the web mid-depth, SF3 increases from section 8 to section 6 (see Figure 2.20(c)).

2.1.6 Corrugation Torsion Stress Resultants

Next, stress resultants related only to corrugation torsion are studied. Some results of this study are used later to develop a corrugation torsion model. Corrugation torsion stress resultants at an arbitrary cross section are shown in Figure 2.21. The values of these stress resultants were calculated based on the FE analysis results.

To get the stress resultants related only to corrugation torsion, the stresses related to St. Venant torsion are eliminated. The section moment $SM_3$ defined in Equation (2.4) is the torque attributed to St. Venant torsion and it exists in both CWGs and FWGs under uniform torsion. $SM_3$ is not considered in the calculation of $M_{bfx}$, $M_{tfx}$, and $M_{wx}$, which are described below. Flange through thickness shear section force, SF4, also exists in the uniform torsion of both CWGs and FWGs. A comparison of the shear section force SF4 at the cross section at the center of an inclined fold is shown in Figure 2.22. It can be seen that for the FWG, SF4 is significant only at locations close to the flange edges, which agrees with St. Venant torsion theory. At the flange edges, SF4 for the CWG is almost the same as that for the FWG, which indicates that it also is related to St. Venant torsion. The difference in SF4 between the CWG and the FWG occurs closer to the middle of the flange, away from the edges, and is related to corrugation torsion. SF4 in this area is small for the CWG and very small for the FWG. Only the SF4 difference is used to calculated $M_{bfx}$, $M_{tfx}$, and $M_{wx}$.

Figure 2.23(a) illustrates the calculation of the total normal force on a bottom flange cross section from nodal values of the section force SF1. SF1 is assumed to be uniform from one node to halfway to the adjacent nodes, so that the normal force for the two edge nodes (node 1 and $n$) is $SF1 \cdot l_e/4$ and $SF1_n \cdot l_e/4$ respectively, and the normal force for an interior node $i$ is $SF1_i \cdot l_e/2$, where $l_e$ is the length of one element. The total normal force on the cross section is
where \( bf \) represents the bottom flange. The moment about an arbitrary point B for SF1 is calculated as the sum of the elemental moments caused by the elemental normal forces

\[
M_{bfz} = -\frac{l_c}{4} \left( SF1_1 \cdot y_1 + SF1_n \cdot y_n \right) - \frac{l_c}{2} \sum_{i=2}^{n-1} SF1_i \cdot y_i
\]

where \( y_i \) is the distance from node \( i \) to point B. Similarly, other stress resultants are determined by the following equations

\[
V_{bfy} = \frac{l_c}{4} \left( SF3_1 + SF3_n \right) + \frac{l_c}{2} \sum_{i=2}^{n-1} SF3_i
\]

\[
V_{bfz} = \frac{l_c}{4} \left( SF4_1 + SF4_n \right) + \frac{l_c}{2} \sum_{i=2}^{n-1} SF4_i
\]

\[
M_{bfy} = \frac{l_c}{4} \left( SF4_1 \cdot y_1 + SF4_n \cdot y_n \right) + \frac{l_c}{2} \sum_{i=2}^{n-1} SF4_i \cdot y_i
\]

\[
M_{bfy} = -\frac{l_c}{4} \left( SM1_1 + SM1_n \right) - \frac{l_c}{2} \sum_{i=2}^{n-1} SM1_i
\]

The following relations are observed from the symmetries shown in Figure 2.16

\[
SF1_{yf} = -SF1_{bf}
\]

\[
SF3_{yf} = -SF3_{bf}
\]

\[
SF4_{yf} = SF4_{bf}
\]

\[
SM1_{yf} = SM1_{bf}
\]

where \( bf \) represents the bottom flange and \( tf \) represents the top flange. The stress resultants on the top flange can then be related to those on the bottom flange by

\[
N_{tx} = -N_{tx}
\]

\[
V_{ty} = -V_{bfy}
\]

\[
V_{tz} = V_{bfz}
\]

\[
M_{tx} = M_{bfz}
\]

\[
M_{ty} = M_{bfy}
\]

\[
M_{tz} = -M_{bfz}
\]

The stress resultants for the flanges are in the global directions since the local and global directions coincide for the flanges as shown in Figure 2.3. This is not the case for the web. For the web, the stress resultants in the local directions are calculated first and the stress resultants in the global directions are then calculated from those in the local directions. The equations for the web stress resultants in the local directions are similar to those for the flange stress resultants given above. Since the web section forces SF1 and SF4 are anti-symmetric about the web mid-depth, stress resultants \( N_{wx} \) and \( V_{wy} \) are zero. The web plate bending stress resultant \( M_{wz} \) is neglected. Web
vertical shear $V_{wz}$ is the same in both the global and local directions and is determined by

$$V_{wz} = \frac{l}{4} \left( SF3_i + SF3_n \right) + \frac{l}{2} \sum_{i=2}^{n} SF3_i$$

The global moments $M_{wx}$ and $M_{wy}$ are calculated from the local moments $M_{w1}$ and $M_{w3}$, which are determined by

$$M_{w1} = \frac{l}{4} \left( SF4_1 \cdot z_1 + SF4_n \cdot z_n \right) + \frac{l}{2} \sum_{i=2}^{n} SF4_i \cdot z_i$$

$$M_{w3} = \frac{l}{4} \left( SF1_1 \cdot z_1 + SF1_n \cdot z_n \right) + \frac{l}{2} \sum_{i=2}^{n} SF1_i \cdot z_i$$

where $z_i$ is the distance between node $i$ and the web mid-depth. The directions of $M_{wx}$ and $M_{wy}$ are defined in Figure 2.21. For the longitudinal fold, $M_{wx}$ and $M_{wy}$ are determined by (see Figure 2.23(b))

$$M_{wx} = M_{w1}$$

$$M_{wy} = -M_{w3}$$

(2.5)

For the inclined fold between section 3 and 5 (see Figure 2.15), $M_{wx}$ and $M_{wy}$ are determined by (see Figure 2.23(c))

$$M_{wx} = M_{w1x} - M_{w3x} = M_{w1} \cos(\alpha) - M_{w3} \sin(\alpha)$$

$$M_{wy} = -M_{w1y} - M_{w3y} = -M_{w1} \sin(\alpha) - M_{w3} \cos(\alpha)$$

(2.6)

For the inclined fold between either section 0 and 1 or section 7 and 8, $M_{wx}$ and $M_{wy}$ are determined by (see Figure 2.23(d))

$$M_{wx} = M_{w1x} + M_{w3x} = M_{w1} \cos(\alpha) + M_{w3} \sin(\alpha)$$

$$M_{wy} = M_{w1y} - M_{w3y} = M_{w1} \sin(\alpha) - M_{w3} \cos(\alpha)$$

(2.7)

The bottom flange stress resultants for the fifth corrugation are shown in Figure 2.24. Positions of critical sections are also shown and are identified by text boxes. The following observations are made:

- $N_{fx}$ is maximum at the centers of the inclined folds and is zero at the centers of the longitudinal folds. So the flange is stretched and compressed alternately between the centers of the longitudinal folds.
- $V_{fy}$ is maximum at the centers of the inclined folds and is almost constant over the longitudinal folds.
- $V_{fs}$ increases linearly from the centers of the inclined folds to the adjacent foldlines and decreases towards the centers of the longitudinal folds.
- $M_{fy}$ is maximum at the centers of the inclined folds and zero at the centers of the longitudinal folds. The bending changes direction at the center of each longitudinal fold.
• $M_{fz}$ shifts away from the zero baseline. Later in the discussion, the overall shift is neglected for the proposed corrugation torsional model so that $M_{fz}$ is zero at the center of each inclined fold.

• It can be seen that symmetry exists for all the stress resultants. If the value of stress resultant on a quarter of a corrugation is known, the value of the stress resultant at other locations can be predicted by symmetry. This is consistent with the observations from the section force plots.

Web stress resultants for the fifth corrugation are shown in Figure 2.25. Positions of critical sections are also shown and are identified by text boxes. Comparing with $V_{fz}$ shown in Figure 2.24(c), it can be seen that the web vertical shear $V_{wz}$ is about twice the flange vertical shear $V_{fz}$ but with opposite signs. The web torque $M_{wx}$ and the bending moment $M_{wy}$ in the global direction are also shown. It can be seen that $M_{wx}$ is very small over the longitudinal fold. $M_{wx}$ is large on the inclined fold, which is due to the contribution of $M_{w3}$. Similar to the flange stress resultants described above, symmetry exists for the web stress resultants.

2.1.7 Corrugation Torsion Resistance Mechanism

According to the previous discussion, CWG uniform torsion resistance is considered to be the sum of the St. Venant torsion resistance and the corrugation torsion resistance. In this section, corrugation torsion at two typical cross sections is studied.

From the FE analysis of the prototype CWG under a twist of 0.05 rad., the total reaction torque, $T$, is 18160 kN-mm. The reaction torque of the comparable FWG, $T_f$, is 10140 kN-mm. The difference is attributed to the torque for corrugation torsion, $cT$, which is 8020 kN-mm. The first cross section studied is section 8 (see Figure 2.15), which is at the center of an inclined fold. Figure 2.26(a) shows the internal forces related to corrugation torsion on this cross section. The corrugation torsion resistance $T_{c8}$ equals

$$T_{c8} = V_{f8}h + 2 \cdot M_{f8x} + M_{wx8}$$

Substituting $V_{f8}$, $M_{f8x}$ and $M_{wx8}$ from the FE analysis results of the prototype CWG

$$T_{c8} = 5.0 \times 1600 + 2 \times 658 - 1115 = 8201 \text{ (kN-mm)}$$

which is nearly equal to the value of $T_c$ given earlier. At section 8,

$$T_{c8} \approx V_{f8}h = 8000 \text{ (kN-mm)}$$

because $M_{f8x}$ and $M_{wx8}$ are opposite in direction and nearly cancel each other.

The second cross section studied is just to the right of the longitudinal fold center labeled section 6 in Figure 2.15 (25 mm from section 6). For convenience, the stress resultants for this cross section are labeled as if they are at section 6, as shown in Figure 2.26(b). The corrugation torsion resistance at this cross section is calculated by taking moments about one of the flange centroids.
\[ T_{c6} = V_{f6}h + V_{wz6} \frac{h}{2} + 2 \cdot M_{fx6} + M_{wx6} \]

where \( h_r \) is the corrugation depth. Substituting \( V_{f6}, V_{wz6}, M_{fx6} \) and \( M_{wx6} \) from the FE analysis results of the prototype CWG

\[ T_{c6} = 1.82 \times 1600 + \frac{300}{2} \times 24 + 2 \times 759 - 3.9 = 8026 \text{ (kN-mm)} \]

\( T_{c6} \) is nearly equal to \( T_c \). Figure 2.25(b) shows that at a cross section near the center of longitudinal fold, \( M_{wx} \approx 0 \).

For any cross section in between the center of the inclined fold and the center of the longitudinal fold, it can be expected that corrugation torsion resistance is due to contributions from \( V_{fx}, V_{wz}, M_{fx} \) and \( M_{wx} \).

### 2.2 3D Static Formulation

The complex state of stresses and stress resultants in a CWG under uniform torsion has been shown above. In this section, a 3D static analysis of the relevant stress resultants is made as an attempt to model corrugation torsion.

The results given previously suggest that one quarter of the length of one corrugation (e.g., from section 6 to 8, see Figure 2.15) serves as the basic unit of a CWG for the analysis of stresses or stress resultants from corrugation torsion due to the apparent symmetry. Three cross sections are arbitrarily selected, e.g. section 6, 7 and 8. Generally, at each of these cross sections, there are 18 unknowns, as illustrated in Figure 2.21, so in total there are 54 unknowns. The stress results on the top flange are related to those on the bottom flange by

\[ \begin{align*}
N_{bfx} &= -N_{fx} = N_{fx} \\
V_{bfy} &= -V_{fy} = V_{fy} \\
V_{bfz} &= V_{tz} = V_{fx} \\
M_{bfz} &= M_{tfx} = M_{fx} \\
M_{bfy} &= M_{tfy} = M_{fy} \\
M_{bfz} &= -M_{tzf} = M_{fx} 
\end{align*} \]

Equation (2.8) eliminates 6 unknowns at each section, and make the total number of unknowns equal to 36. Three free bodies can be considered: from section 0 to section 6, from section 0 to section 7, and from section 0 to section 8 (see Figure 2.15). At section 0, the total stress resultant is the corrugation torsion resistance \( T_c \).

Six static equilibrium equations can be found for the free body from section 0 to section 6.
\[ N_{fx6} + N_{fy6} + N_{wx6} = 0 \]
\[ V_{bfy6} + V_{ty6} + V_{wy6} = 0 \]
\[ V_{bfz6} + V_{t fz} + V_{wz6} = 0 \]
\[ M_{bfy6} + M_{tfy6} + M_{wy6} + \left( V_{bfy6} - V_{ty6} \right) \frac{h}{2} + V_{wz6} \frac{h}{2} = T_c \quad (2.9) \]
\[ M_{bfy6} + M_{tfy6} + M_{wy6} + \left( N_{bfy6} - N_{ty6} \right) \frac{h}{2} = 0 \]
\[ M_{bfz6} + M_{t fz} + M_{wz6} = 0 \]

Substituting Equation (2.8) into Equation (2.9) results in
\[ N_{wx6} = 0 \]
\[ V_{wy6} = 0 \]
\[ 2V_{fx6} + V_{wz6} = 0 \quad (2.10) \]
\[ 2M_{fx6} + M_{wx6} + V_{fy6} h + V_{wz6} \frac{h}{2} = T_c \]
\[ 2M_{fx6} + M_{wy6} + N_{fx6} h = 0 \]
\[ M_{wz6} = 0 \]

Similar equations can be written for the two free bodies from section 0 to section 7 and from section 0 to section 8. Now at each cross section, the three stress resultants, \( N_{wx} \), \( V_{wy} \) and \( M_{wz} \), are known to be zero. At each cross section, nine unknowns remain which are \( N_{fx} \), \( V_{fy} \), \( V_{fz} \), \( M_{fx} \), \( M_{fy} \), \( M_{fz} \), \( V_{wz} \), \( M_{wx} \) and \( M_{wy} \). So the total number of unknowns is 27. At each of the three cross sections, there are three equilibrium equations.
\[ 2V_{fx} + V_{wz} = 0 \]
\[ 2M_{fx} + M_{wx} + V_{fy} h + V_{wz} \frac{h}{2} = T_c \quad (2.11) \]
\[ 2M_{fx} + M_{wy} + N_{fx} h = 0 \]

So the total number of static equilibrium equations is 9. This problem is a highly statically indeterminate problem.

From the above equations, \( V_{wz} = -2V_{fx} \) which agrees with the observations from Figure 2.24(c) and Figure 2.25(a). Other unknowns can be eliminated based on the FE analysis results. Figure 2.24 shows that
\[ N_{fx6} = 0 \]
\[ M_{fx6} = 0 \]
\[ M_{fx6} = 0 \]

the following equation was given in Equation (2.10)
\[ 2M_{fx6} + M_{wy6} + N_{fx6} h = 0 \]
and substituting $N_{f6} = 0$ and $M_{f6} = 0$ into this equation shows that $M_{wy6} = 0$. Figure 2.24 also shows that 
\[ V_{f8} = 0 \]
\[ M_{f8} = 0 \]

The following equation was given in Equation (2.11) 
\[ 2V_{f8} + V_{w8} = 0 \]
and substituting $V_{f8} = 0$ into this equation shows that $V_{w8} = 0$. So the total number of unknowns is reduced by 7 and total number of static equilibrium equations is reduced by 2. As a result, the total number of unknowns is 20 and the total number of equations is 7.

Next, two free bodies, shown in Figure 2.27, are considered. Free body A which is from section 6 to section 7 is shown in Figure 2.27(a). Free body B which is from section 7 to section 8 is shown in Figure 2.27(b). In these figures, free bodies A and B are further divided into smaller free bodies by separating the flanges and web where the results from above analysis have been incorporated. New unknowns are introduced and new static equilibrium equations can be formulated. For free body A, which is shown in Figure 2.27(a), six equilibrium equations can be written for each of the bottom flange, web, and top flange plates. For the bottom flange, the six equilibrium equations are
\[ V_{whx4} + N_{f7} = 0 \]
\[ -V_{f6} + V_{wh4} + V_{f7} = 0 \]
\[ -V_{f6} + N_{whx4} + V_{f7} = 0 \]
\[ -M_{f6} + M_{whx4} + M_{f7} + N_{wbc4} \frac{h_y}{2} = 0 \]
\[ -M_{wbc4} - M_{f7} - \left( V_{f6} + V_{f7} \right) \frac{b}{4} = 0 \]
\[ M_{wbc4} + M_{f7} + \left( V_{f6} + V_{f7} \right) \frac{b}{4} - V_{whx4} \frac{h_y}{2} = 0 \]

where moments are taken about bottom flange centroid. For the web, the six equilibrium equations are
\[ -V_{wbc4} + V_{wtx4} = 0 \]
\[ -V_{wbc4} + V_{wcy4} = 0 \]
\[ -V_{w6} - N_{wbc4} + N_{wz4} + V_{w7} = 0 \]
\[ -M_{w6} - M_{wbc4} + M_{wz4} + M_{w7} - \left( V_{wbc4} + V_{wcy4} \right) \frac{D}{2} = 0 \]
\[ M_{wbc4} - M_{wcy4} - M_{wz4} - \left( V_{wbc4} + V_{wtx4} \right) \frac{D}{2} - \left( V_{w6} + V_{w7} \right) \frac{b}{4} = 0 \]
\[ -M_{wbc4} + M_{wz4} = 0 \]
where moments are taken about the web fold center. For the top flange, the six equilibrium equations are

\[-V_{wtx4} - N_{fx7} = 0\]
\[V_{fy6} - V_{wty4} - V_{fy7} = 0\]
\[-V_{fz6} - N_{wz4} + V_{fz7} = 0\]
\[-M_{fx6} - M_{wtx4} + M_{fx7} - N_{wz4} \frac{h_r}{2} = 0\]  \hspace{1cm} (2.14)
\[M_{wty4} - M_{fy7} - \left( V_{fy6} + V_{fy7} \right) \frac{b}{4} = 0\]
\[-M_{wz4} - M_{fz7} - \left( V_{fz6} + V_{fz7} \right) \frac{b}{4} + V_{wtx4} \frac{h_r}{2} = 0\]

In these equations, the positive force is in the positive global axis direction and the positive moment is about the positive global axis by the right-hand-rule. 12 new unknowns are introduced which are the stress resultants at the web-bottom flange interface \( V_{wbc4}, V_{wyd4}, N_{wbca}, M_{wbn4}, M_{wyb4}, \) and \( M_{wbd4} \) and the corresponding stress resultants at the web-top flange interface \( V_{wta4}, V_{wyd4}, N_{wza4}, M_{wta4}, M_{wyd4}, \) and \( M_{wzd4} \).

From Equation (2.13) and FE observations, the follow equations are obtained

\[V_{wbc4} = V_{wtx4} = V_{wtx4}\]
\[V_{wyd4} = V_{wty4} = V_{wyd4}\]
\[N_{wbca} = -N_{wz4} = N_{wz4}\]
\[M_{wbn4} = -M_{wtx4} = M_{wtx4}\]
\[M_{wyb4} = -M_{wyd4} = M_{wyd4}\]
\[M_{wbd4} = M_{wzd4} = M_{wzd4}\]  \hspace{1cm} (2.15)

Substituting Equation (2.15) to Equations (2.12), (2.13) and (2.14) results in

\[-V_{fy6} + V_{wyd4} + V_{fy7} = 0\]
\[-V_{fz6} + V_{wzd4} + V_{fz7} = 0\]
\[-M_{fx6} + M_{wtx4} + M_{fx7} + N_{wz4} \frac{h_r}{2} = 0\]  \hspace{1cm} (2.16)
\[-M_{wyd4} - M_{fy7} - \left( V_{fy6} + V_{fy7} \right) \frac{b}{4} = 0\]
\[M_{wzd4} + M_{fz7} + \left( V_{fz6} + V_{fz7} \right) \frac{b}{4} - V_{wtx4} \frac{h_r}{2} = 0\]
\[-V_{wz6} - 2N_{wz4} + V_{wz7} = 0\]
\[-M_{wz6} - 2M_{wtx4} + M_{wz7} - V_{wyd4}D = 0\]  \hspace{1cm} (2.17)
\[2M_{wyd4} - M_{wy7} + V_{wyd4}D - \left( V_{wz6} + V_{wz7} \right) \frac{b}{4} = 0\]
\[-V_{wx4} - N_{fx7} = 0 \]
\[V_{fy6} - V_{wy4} - V_{fy7} = 0 \]
\[V_{fz6} + N_{wz4} + V_{fz7} = 0 \]
\[-M_{fx6} + M_{wx4} + M_{fx7} + N_{wz4} \frac{h_r}{2} = 0 \]  \hspace{1cm} (2.18)
\[-M_{wyd} - M_{fy7} - (V_{fz6} + V_{fz7}) \frac{b}{4} = 0 \]
\[-M_{wzA} - M_{fy7} - (V_{fy6} + V_{fy7}) \frac{b}{4} + V_{wxA} \frac{h_r}{2} = 0 \]

It can be seen that Equation (2.16) and Equation (2.18) are equivalent. Equation (2.17) can be derived from Equation (2.11) and (2.16) by neglecting the difference between \(h\), the distance between the centroids of the top and bottom flanges, and \(D\), the web depth.

In summary, from free body A, six new unknowns are introduced and they are \(V_{wx4}, V_{wy4}, N_{wz4}, M_{wx4}, M_{wyd}\) and \(M_{wz4}\). There are also six independent static equilibrium equations introduced, given by Equation (2.16).

Similarly, from free body B, six unknowns are introduced and they are \(V_{wxB}, V_{wyB}, N_{wzB}, M_{wxB}, M_{wyd}\) and \(M_{wzB}\). Also, six independent static equilibrium equations are introduced.
\[-N_{fx7} + V_{wxB} = 0 \]
\[-V_{fy7} + V_{wyB} + V_{fy8} = 0 \]
\[-V_{fz7} + N_{wzB} = 0 \]
\[-M_{fx7} + M_{wxB} + M_{fx8} + N_{wzB} \frac{h_r}{4} = 0 \]  \hspace{1cm} (2.19)
\[M_{fy7} - M_{wyB} - M_{fy8} - V_{fy7} \frac{d}{4} = 0 \]
\[-M_{fz7} + M_{wxB} + (V_{fy7} + V_{fy8}) \frac{d}{4} - V_{wxB} \frac{h_r}{4} = 0 \]

In summary, from free body A and B, 12 new unknowns are introduced and 12 new static equilibrium equations are introduced. The total number of unknowns is now 32 and the total number of equilibrium equations is now 19. To solve this problem, appropriate assumptions are made to reduce the number of unknowns.

2.3 Proposed Corrugation Torsion Model

Section 2.1.5 shows that the stresses and section forces from FE analysis of a CWG under uniform torsion are very complicated, which makes the static model described in Section 2.1.4 too simple and the 3D static formulation introduced in Section 2.2 too complex to use. In this section, a corrugation torsion model is proposed, which is based on stresses closer to but simpler than those from the FE analysis results.
2.3.1 Assumed Forces between Flange and Web

The variation of the flange bending moment $M_{f_y}$ and vertical shear $V_{f_z}$ is caused by the variation of the normal force between the flanges and the web. These normal force can be observed as the normal section force in the web ($SF_2$) at the web-flange interface. $SF_2$ in the web at the web-bottom flange interface from the FE analysis is shown in Figure 2.28. Positions of critical sections are also shown and are identified by text boxes. It can be seen that $SF_2$ is almost constant over the inclined folds. It reverses direction at the foldline and is linearly distributed over the longitudinal fold with a value of zero at the center of the longitudinal fold. The distribution of $SF_2$ causes the flange vertical shear $V_{f_z}$ shown in Figure 2.24(c). Also shown in Figure 2.28 is the shear section force $SF_3$ between the web-bottom flange interface from the FE analysis. It can be seen that the maximum values of $SF_3$ are at the foldlines and for half of the corrugation, $SF_3$ is in one direction and for the other half of the corrugation, $SF_3$ is in the other direction. The distribution of $SF_3$ causes the flange normal force $N_{f_x}$ shown in Figure 2.24(a). Within the length of one corrugation, $SF_2$ and $SF_3$ are symmetric about section 4 and for half of the corrugation, they are symmetric about section 2 (or section 6).

To understand the variation of the flange bending moment $M_{f_y}$ and the vertical shear $V_{f_z}$ along the girder, the normal force between flanges and web is considered. Based on the FE analysis results for $SF_2$ in the web shown above, the assumed normal section force $n$ between the bottom flange and the web is shown in Figure 2.29(a). $n$ is assumed to be constant with a magnitude of $n_i$ over the inclined fold. The direction of $n$ is reversed at the foldline and is linearly distributed over the longitudinal fold with a magnitude of zero at the center of the longitudinal fold. The maximum value of $n$ on the longitudinal fold is labeled as $n_i$. At the foldline on the other end of the longitudinal fold, the direction of $n$ is reversed again so that the directions of $n$ on the two adjacent inclined folds are opposite to each other. Figure 2.29(a) also shows the shear forces between the flange and the web. The magnitudes of shear force on the longitudinal fold and on half of the inclined fold are $V_{wl}$ and $V_{wi}$, respectively. As shown later, $V_{wl}$ and $V_{wi}$ are functions of $n_i$ and $n_j$.

The number of unknowns between the flange and the web, discussed in Section 2.2, can be further reduced by assuming that:
- The normal and shear stresses are constant over the thickness of the web.
- $n_{i}$, $n_{j}$, $V_{wf}$ and $V_{wi}$ are the only forces between the flanges and the web.

Using these assumptions, for the free body A shown in Figure 2.27(a), $V_{wyl}$, $M_{wxd}$ and $M_{wzd}$ are zero and $V_{wx}$, $N_{wzd}$ and $M_{wyd}$ are all functions of $n_{i}$ and $n_{j}$. For the free body B shown in Figure 2.27(b), $M_{wxb}$, $M_{wyb}$ and $M_{wzb}$ are zero and $V_{wxB}$,
and \( w \) are all functions of \( n_i \) and \( n_j \). Since now \( n_i \) and \( n_j \) are the only two unknown section forces acting at the web-flange interface, the total number of unknowns defined in Section 2.2 is reduced to 22. The total number of static equilibrium equations is unchanged and is still 19. The problem now is statically indeterminate to the third degree.

### 2.3.2 Static Equilibrium Equations

For convenience, the static equilibrium equations presented in Section 2.2 are reformulated in this section. From the in-plane moment equilibrium of the bottom flange, shown in Figure 2.29(a),

\[
V_{y_2} L_c - V_{w_l} h_r - V_{w_l} L_c \sin(\alpha) = 0
\]  

(2.20)

where \( L_c \) is the projected axial length of one corrugation. Figure 2.29(b) shows the stress and stress resultants on the bottom flange between section 6 and section 7. From the force equilibrium in the three global directions,

\[
N_{x_7} + 0.5V_{w_l} = 0
\]  

(2.21)

\[-V_{y_6} + V_{y_7} = 0\]

(2.22)

\[-V_{z_6} + V_{z_7} + F_z(n_i) = 0\]

(2.23)

where

\[F_z(n_i) = \frac{n_i \cdot b}{4}\]

From the moment equilibrium about the three global axes and by taking moment about point \( m \),

\[-M_{x_6} + M_{x_7} + M_{x_m}(n_i) = 0\]

(2.24)

\[V_{y_7} \frac{b}{2} + M_{y_7} + M_{y_m}(n_i) = 0\]

(2.25)

\[V_{z_7} \frac{b}{2} + M_{z_7} - \frac{V_{w_l}}{2} \frac{h_r}{2} = 0\]

(2.26)

where

\[M_{x_m}(n_i) = F_z(n_i) \cdot \frac{h_r}{2} = \frac{1}{8} n_i \cdot b \cdot h_r\]

\[M_{y_m}(n_i) = F_z(n_i) \cdot \frac{b}{3} = \frac{1}{12} n_i \cdot b^2\]

Equations (2.21) to (2.26) are a version of Equation (2.16) obtained by using the assumed section force distribution at the web-flange interface and by taking moments about the reference point \( m \).

Figure 2.29(c) shows the stresses and stress resultants on the bottom flange between section 7 and section 8. From the force equilibrium in the three global directions,

\[-N_{y_7} + N_{y_8} + V_{w_l} \cos(\alpha) = 0\]

(2.27)

\[-V_{y_7} + V_{y_8} - V_{w_l} \sin(\alpha) = 0\]

(2.28)
\[ -V_{f7} - F_z(n_i) = 0 \] (2.29)

where

\[ F_z(n_i) = \frac{n_i \cdot d}{2 \cdot \cos(\alpha)} \]

From the moment equilibrium about the three global axes and by taking moment about point \( n \),

\[ -M_{f7} + M_{f8} - M_{xn}(n_i) = 0 \] (2.30)

\[ M_{f7} - M_{f8} - V_{f7} \frac{d}{2} - M_{yn}(n_i) = 0 \] (2.31)

\[ V_{f7} \frac{d}{2} - M_{f7} = 0 \] (2.32)

where

\[ M_{xn}(n_i) = F_z(n_i) \cdot \frac{h_x}{4} = \frac{n_i \cdot d \cdot h_x}{8 \cdot \cos(\alpha)} \]

\[ M_{yn}(n_i) = F_z(n_i) \cdot \frac{d}{4} = \frac{n_i \cdot d^2}{8 \cdot \cos(\alpha)} \]

Equations (2.27) to (2.32) are a version of Equation (2.19) obtained by using the assumed section force distribution at the web-flange interface and by taking moments about the reference point \( n \).

Figure 2.29(d) shows the stresses and stress resultants on the longitudinal web fold between section 6 and section 7. From the force equilibrium in the vertical direction,

\[ -V_{w6} + V_{w7} - 2F_z(n_i) = 0 \] (2.33)

From the moment equilibrium about global axis X and Y and by taking moment about point \( p \),

\[ -M_{wx6} + M_{wx7} = 0 \] (2.34)

\[ V_{w7} \frac{b}{2} + M_{wy7} - \frac{V_{wy7}}{2} D - 2M_{yn}(n_i) = 0 \] (2.35)

Equations (2.33) to (2.35) are a version of Equation (2.17) obtained by using the assumed section force distribution at the web-flange interface and by taking moments about the reference point \( p \).

Figure 2.29(e) shows the stresses and stress resultants on the inclined fold between section 7 and section 8. From the force equilibrium in the vertical direction,

\[ -V_{w7} + 2F_z(n_i) = 0 \] (2.36)

From the moment equilibrium about global axis X and Y and by taking moment about point \( q \),

\[ -V_{w7} \frac{h_x}{4} - M_{wx7} + M_{wx8} + V_{wy7} \sin(\alpha) \cdot D = 0 \] (2.37)

\[ V_{w7} \frac{d}{4} - M_{wy7} + M_{wy8} - V_{wy7} \cos(\alpha) \cdot D = 0 \] (2.38)
Additional equilibrium equations are from Equation (2.11) for section 6, section 7 and section 8.

\[ 2V_{f6} + V_{wz6} = 0 \]
\[ 2V_{f7} + V_{wz7} = 0 \]
\[ 2M_{fx6} + M_{wx6} + V_{f6}h + V_{wz6} \frac{h}{2} = T_c \]
\[ 2M_{fx7} + M_{wx7} + V_{f7}h + V_{wz7} \frac{h}{2} = T_c \]
\[ 2M_{fx8} + M_{wx8} + V_{f8}h = T_c \]
\[ 2M_{fy7} + M_{wy7} + N_{f7}h = 0 \]
\[ 2M_{fy8} + M_{wy8} + N_{f8}h = 0 \]

26 equations are listed above, but not all of them are independent as discussed in Section 2.2. For example, Equations (2.20), (2.33) to (2.38) can all be derived from other equations so that the total number of independent equations is 19. The total number of unknowns is 22.

2.3.3 Solution Approach

The solution of the above static equilibrium equations for the corrugation torsion model is in concept similar to a solution approach that could be used to determine the forces and moments for the beam shown in Figure 2.30. This beam is statically indeterminate to the first degree. Instead of employing a deformation compatibility condition, which is part of the classical solution for such a problem, \( R_2 = \lambda \cdot R_1 \) is assumed and the unknown forces and moments are expressed as a function of \( \lambda \).

\[ R_1(\lambda) = \frac{1}{1 + \lambda} P \]
\[ R_2(\lambda) = \frac{\lambda}{1 + \lambda} P \]
\[ M_1(\lambda) = \left( \alpha - \frac{\lambda}{1 + \lambda} \right) P \]  

(2.46)

Next the beam is analyzed using FE analysis, \( R_1 \) is determined from the FE analysis as \( R_{1,FE} \), and \( \lambda \) is solved by equating \( R_1(\lambda) = R_{1,FE} \).

The analysis is then repeated, \( \lambda \) is determined for a practical range of indeterminate beams. Then a regression analysis of \( \lambda \) is made as a function of the beam bending stiffness \( EI \), the span \( L \), and the span parameter \( \alpha \). For future beam analysis problem, \( \lambda \) is determined from the regression equation, and all of the internal forces are determined from Equation (2.46). The equilibrium equations of the static corrugation torsion model are solved using a similar approach.
2.4 Static Solution for CWG Corrugation Torsion Model

Two static solutions were derived for the proposed corrugation torsion model discussed in Section 2.3. One solution was developed based on the assumption that the shear flow is uniform over the web depth. A second solution was developed based on the assumption that the shear flow is non-uniform over the web depth. The results of the second solution are not as good as the results of the first solution compared to the FE analysis results. Here, only the solution assuming a constant shear flow over the web depth is presented.

2.4.1 Section Force Distributions on the Web Folds

The web vertical shear $V_{wz}$ can be determined by referring to Figure 2.29(d) and Figure 2.29(e) and knowing from the FE results that $V_{wz8} = 0$.

\[
V_{wz} = \begin{cases} 
  c \cdot n_i - \frac{b \cdot n_i}{2} + \frac{2n_i}{b} x^2 & \text{when } 0 \leq x \leq \frac{b}{2} \\
  2n_i \left( \frac{c}{2} - x_i \right) & \text{when } 0 \leq x_i \leq \frac{c}{2}
\end{cases}
\]

(2.47)

where $x$ is measured from section 6 to section 7 in the global axis $X$ direction and $x_i$ is measured from section 7 to section 8 in the inclined fold direction. Section 2.1.5 shows that the region of the CWG between section 6 and section 8 is the basic unit for the stresses and section forces from uniform torsion. $V_{wz}$ at other locations can be determined using symmetry. Figure 2.19 shows that the web section forces are symmetric about the web mid-depth so that only the section forces for a quarter of a longitudinal fold and a quarter of an inclined fold need to be studied, for example for the web portion identified by $ABED$ and $BCFE$ shown in Figure 2.31(a). Since the shear flow $f$ is assumed to be constant over the web depth, it is simply equal to $V_{wz}$ divided by the web depth $D$.

\[
f = \frac{1}{D} \begin{cases} 
  c \cdot n_i - \frac{b \cdot n_i}{2} + \frac{2n_i}{b} x^2 & \text{when } 0 \leq x \leq \frac{b}{2} \\
  2n_i \left( \frac{c}{2} - x_i \right) & \text{when } 0 \leq x_i \leq \frac{c}{2}
\end{cases}
\]

(2.48)

Now the free body $abcd$ within $ABED$ shown in Figure 2.31(b) is studied. The body $abcd$ is of arbitrary size within $ABED$, defined by the local variables $x$ and $\xi$ and $z$ and $\zeta$. The section forces on free body $abcd$ are shown in Figure 2.31(c) where $n_i$ and $n_z$ are the web normal section force in the local 1 and 2 directions. $f_1$ is the shear flow on edge $ad$ and $bc$ and $f_2$ is the shear flow on edge $ab$ and $cd$. Static equations are written for the force equilibrium in local 1 and 2 directions.

\[
-\int_0^x n_i(0, \zeta) d\zeta + \int_0^x f_2(\xi, \zeta) d\xi - \int_0^\xi f_2(\xi, 0) d\xi + \int_0^\zeta n_i(x, \zeta) d\zeta = 0
\]

\[
-\int_0^\xi n_z(\xi, 0) d\xi - \int_0^\xi f_i(0, \zeta) d\zeta + \int_0^\xi f_i(x, \zeta) d\zeta + \int_0^\zeta n_z(\xi, z) d\zeta = 0
\]

(2.49)
Figure 2.20(a) and Figure 2.20(b) show that
\[ n_1(0, \zeta) = 0 \]
\[ n_2(\xi, 0) = 0 \]
Equation (2.48) shows shear flow only depends on \( \xi \) so that
\[ f_2(\xi, z) = f_2(\xi, 0) \]
\( f_1(0, \zeta) \) and \( f_1(x, \zeta) \) are independent of \( \zeta \). Equation (2.49) can be rewritten as
\[
\int_0^\infty n_1(x, \zeta)d\zeta = 0 \\
- f_1(0, \zeta) \cdot z + f_1(x, \zeta) \cdot z + \int_0^\infty n_2(\xi, z)d\xi = 0
\]
To satisfy above equation for an arbitrary \( z \),
\[ n_1(x, \zeta) = 0 \quad (2.50) \]
So, \( n_1 \) is zero at any location on the longitudinal fold. From the second equation
\[
\int_0^\infty n_2(\xi, z)d\xi = f_1(0, \zeta) \cdot z - f_1(x, \zeta) \cdot z
\]
Taking the derivative with respect to \( x \) results in
\[ n_2(x, z) = \frac{d}{dx} \int_0^\infty n_2(\xi, z)d\xi = \frac{d}{dx} \left( f_1(0, \zeta) \cdot z - f_1(x, \zeta) \cdot z \right) \quad (2.51) \]
Substituting Equation (2.48) into (2.51) and taking the derivative results in
\[ n_2(x, z) = -\frac{x}{b/2} \cdot \frac{z}{D/2} \cdot n_1 = -\frac{4 \cdot n_1}{b \cdot D} x \cdot z \quad (2.52) \]
Therefore, \( n_2 \) increases linearly from \( AD \) to \( BE \) and for an arbitrary \( x \), \( n_2 \) increases linearly from \( AB \) to \( DE \). So the force equilibrium conditions in the local 1 and 2 directions are satisfied through Equation (2.50) and (2.52). Now check the in-plane moment equilibrium, taking the moment about point \( b \)
\[
\sum M_b = f_1(0) \cdot z \cdot x - \int_0^\infty f_2(\xi) \cdot d\xi \cdot z = \int_0^\infty n_2(\xi, z) \cdot (x - \xi) \cdot d\xi
\]
Substituting \( f_1(0) \), \( f_2(\xi) \) and \( n_2(\xi, z) \) and it is found \( \sum M_b = 0 \) so that the in-plane moment equilibrium is also satisfied.

In summary, the section forces on longitudinal fold \( ABED \) are
\[ n_1(x, z) = 0 \]
\[ n_2(x, z) = -\frac{4 \cdot n_1}{b \cdot D} x \cdot z \quad (2.53) \]
\[ f(x, z) = \frac{1}{D} \left( c \cdot n_1 - \frac{b \cdot n_1}{2} + \frac{2n_1}{b} \cdot x^2 \right) \]
where \( x \) is measured from \( A \) to \( B \) and \( z \) is measured from \( A \) to \( D \) (Figure 2.31(b)) and \( 0 \leq x \leq b/2 \), \( 0 \leq z \leq D/2 \). Similar derivations can be made for the inclined fold, which is also shown in Figure 2.31, and the results are
\[ n_1(x_i, z) = 0 \]
\[ n_2(x_i, z) = \frac{2z}{D} n_i \]
\[ f(x_i, z) = \frac{2n_i}{D} \left( \frac{c}{2} - x_i \right) \]  

(2.54)

where \( x_i \) is measured from \( B \) to \( C \) and \( z \) is measured from \( B \) to \( E \) (Figure 2.31(a)) and \( 0 \leq x_i \leq c/2 \), \( 0 \leq z \leq D/2 \). It can be noted that \( n_1 \) is also zero on the inclined fold. At an arbitrary \( z \), \( n_2 \) is constant and \( n_2 \) increases linearly from \( BC \) to \( EF \).

### 2.4.2 Internal Forces for Corrugation Torsion Model

For the corrugated web, it can be noted that \( N_{wx}, V_{wy} \) and \( M_{wz} \) are zero by Equation (2.10). In the previous section, \( V_{wz} \) is determined and \( n_i \) is shown to be zero so that \( M_{w3} = 0 \). If \( M_{w1} \) is also neglected, then from Equation (2.5) to (2.7), \( M_{wz} \) and \( M_{wy} \) are also zero. In summary, the internal forces on any vertical cut on the web between section 6 and section 8 are

\[
\begin{align*}
N_{wx} &= 0 \\
V_{wy} &= 0 \\
V_{wz} &= \begin{cases} 
    c \cdot n_i - \frac{b \cdot n_i}{2} + \frac{2n_i}{b} x^2, & \text{for longitudinal fold} \\
    2n_i \left( \frac{c}{2} - x_i \right), & \text{for inclined fold} 
\end{cases} \\
M_{wx} &= 0 \\
M_{wy} &= 0 \\
M_{wz} &= 0
\end{align*}
\]  

(2.55)

where \( 0 \leq x \leq b/2 \) and is measured from section 6 to 7 in the direction of the longitudinal fold (see Figure 2.29(d)) and \( 0 \leq x_i \leq c/2 \) and is measured from section 7 to section 8 in the direction of the inclined fold (see Figure 2.29(e)). The internal forces on the other part of web can be determined by symmetry.

The bottom flange internal forces between section 6 and section 8 are determined using Figure 2.32(a) where \( f_z \) is the shear flow between the longitudinal fold and the bottom flange and can be determined from Equation (2.53). \( n_i(x) \) is the normal section force between the longitudinal fold and the bottom flange and is defined as

\[ n_i(x) = \frac{2x}{b} n_i \]

Six static equilibrium equations can be written for the free body shown in Figure 2.32(a) as
\[ V_{lx} + N_{fx}(x) = 0 \]
\[-V_{fy} + V_{fr}(x) = 0 \]
\[-V_{fx} + N_{lx} + V_{fc}(x) = 0 \]
\[-M_{fx} + N_{lx} \frac{h}{2} + M_{fx}(x) = 0 \]
\[-V_{fx}x + N_{lx} \frac{x}{3} - M_{fy}(x) = 0 \]
\[ V_{fy}x - V_{lx} \frac{h}{2} + M_{fz}(x) = 0 \]

where \( N_{lx} \) and \( V_{lx} \) are defined as
\[
N_{lx} = \int_0^x n_i(x) dx = \frac{n_i}{b} x^2
\]
\[
V_{lx} = \int_0^x f_2(x) dx = \frac{x}{6b \cdot D} \left[ (4x^2 - 3b^2) n_i + 6b \cdot c \cdot n_i \right]
\]

where \( 0 \leq x \leq b/2 \). Similarly, six static equilibrium equations can be written for the free body shown in Figure 2.32(b) as

\[-N_{fx} + V_{fx} \cos(\alpha) + N_{fx}(x) = 0 \]
\[-V_{fy} + V_{fc}(x) = 0 \]
\[-V_{fc} - N_{ix} + V_{fc}(x) = 0 \]
\[-M_{fx} - N_{ix} \frac{d-x}{2} \tan(\alpha) + M_{fx}(x) = 0 \]
\[-V_{fx}x + M_{fx} - N_{ix} \frac{x}{2} - M_{fz}(x) = 0 \]
\[ V_{fz}x - M_{fz} - V_{ix} \left( \frac{d}{2} - x \right) \sin(\alpha) + M_{fz}(x) = 0 \]

where \( N_{ix} \) and \( V_{ix} \) are defined as
\[
N_{ix} = -\frac{n_i}{\cos(\alpha)} x
\]
\[
V_{ix} = \int_0^{x / \cos(\alpha)} f_2(x_i) dx_i = \frac{x}{\cos(\alpha)} \frac{c - x / \cos(\alpha)}{D} n_i
\]

where \( 0 \leq x \leq d/2 \). From Equation (2.22) and (2.28),
\[ V_{fz} = V_{fz'} - V_{wi} \sin(\alpha) \]

where \( V_{wi} \) can either be determined from Equation (2.57) with \( x = d/2 \) or from Equation (2.36) and (2.38) note that \( M_{wy7} = M_{wy8} = 0 \),
\[ V_{wi} = \frac{c^2}{4D} n_i \]  

so that
\[ V_{f6} = V_{f8} = \frac{c^2}{4D} n_i \sin(\alpha) \]

From Equation (2.23) and (2.29)
\[ V_{f6} = \frac{b}{4} n_i - \frac{c}{2} n_i \]

From Equation (2.24) and (2.30) and note that \( d = c \cdot \cos(\alpha) \) and \( h_r = c \cdot \sin(\alpha) \),
\[ M_{f6} = M_{f8} = \frac{c \sin(\alpha)}{8} (c \cdot n_i - b \cdot n_i) \]

In summary, the bottom flange internal forces between section 6 and section 7 are
\[ N_f(x) = -\frac{x}{6bD} \left[ (4x^2 - 3b^2) n_i + 6b \cdot c \cdot n_i \right] \]
\[ V_{f6}(x) = V_{f6} \]
\[ V_{f7}(x) = V_{f6} - \frac{n_i}{b} x^2 \]
\[ M_{f6}(x) = M_{f6} = \frac{c \sin(\alpha) n_i}{2b} x^2 \]
\[ M_{f7}(x) = -V_{f6} x + \frac{n_i}{3b} x^3 \]
\[ M_{f8}(x) = -V_{f6} x + \frac{x \cdot c \sin(\alpha)}{12bD} \left[ (4x^2 - 3b^2) n_i + 6b \cdot c \cdot n_i \right] \]

where \( 0 \leq x \leq b/2 \) and is measured from section 6 to section 7 in the global X axis direction (see Figure 2.32(a)) and
\[ V_{f6} = V_{f8} = \frac{c^2}{4D} n_i \sin(\alpha) \]
\[ V_{f6} = \frac{b}{4} n_i - \frac{c}{2} n_i \]
\[ M_{f6} = M_{f8} = \frac{c \sin(\alpha)}{8} (c \cdot n_i - b \cdot n_i) \]

The bottom flange internal forces between section 7 and section 8 are
\[ N_{fc}(x) = N_{fc7} - \frac{c - x / \cos(\alpha)}{D} x \cdot n_i \]
\[ V_{fi}(x) = V_{fi7} + x \tan(\alpha) \frac{c - x / \cos(\alpha)}{D} n_i \]
\[ V_{fi}(x) = V_{fi7} + \frac{n_i}{\cos(\alpha)} x \]
\[ M_{fx}(x) = M_{fx7} + \frac{\tan(\alpha)}{2 \cos(\alpha)} x(d - x) n_i \]
\[ M_{fy}(x) = -V_{fi7} x + M_{fy7} - \frac{x^2}{2 \cos(\alpha)} n_i \]
\[ M_{fz}(x) = -V_{fi7} x + M_{fz7} + x \tan(\alpha) \frac{c - x / \cos(\alpha)}{D} \left( \frac{d}{2} - x \right) n_i \]

where \( 0 \leq x \leq c \cos(\alpha)/2 \) and \( x \) is measured from section 7 to section 8 in the global X axis direction (see Figure 2.32(b)). Substituting \( b/2 \) for \( x \) in Equation (2.59)

\[ N_{fc7} = \frac{b}{6D} \left( b \cdot n_i - 3c \cdot n_i \right) \]
\[ V_{fy7} = V_{fi8} = \frac{c^2 \sin(\alpha)}{4D} n_i \]
\[ V_{fy7} = -\frac{c}{2} n_i \]
\[ M_{fx7} = M_{fx8} = \frac{c^2 \sin(\alpha)}{8} n_i \]
\[ M_{fy7} = \frac{b}{4} \left( \frac{-b}{3} n_i + c \cdot n_i \right) \]
\[ M_{fz7} = -\frac{b}{2} V_{fy8} = \frac{b \cdot c \sin(\alpha)}{24D} \left( 2b \cdot n_i - 9c \cdot n_i \right) \]

### 2.4.3 Corrugation Torsion Solution

From Equations (2.35) and (2.36) with \( M_{wy7} = 0 \) (or by doubling the value of \( V_{fx} \) from Equation (2.56) with \( x = b/2 \)), the total shear at the web-flange interface along the longitudinal fold is

\[ V_{wy} = \frac{b}{3D} \left( 3c \cdot n_i - b \cdot n_i \right) \]  \( (2.61) \)

From Equation (2.43) and equation \( 2M_{fx8} + M_{wx8} = 0 \) according to the discussion in Section 2.1.7,

\[ T_c = V_{fy8} h \]
Substituting \( V_{fy} \) into Equation (2.20),

\[
T_c = \frac{h}{L_c} \left( V_{wi} \cdot h_r + V_{wi} \cdot L_c \cdot \sin(\alpha) \right)
\]  

(2.62)

Substituting \( V_{wi} \) and \( V_{wi} \),

\[
T_c = \frac{h \cdot h_r}{L_c} \left[ \left( b + \frac{L_c}{4} \right) c \cdot n_i - \frac{b^2}{3} \cdot n_i \right]
\]

Now assume

\( n_i = \lambda \cdot n_i \)

Then

\[ T_c = C \cdot n_i \]

where

\[
C = \frac{h \cdot h_r}{D L_c} \left[ \left( b + \frac{L_c}{4} \right) c - \frac{b^2}{3} \cdot \lambda \right]
\]

Then

\[ n_i = \frac{T_c}{C} \]

(2.63)

The corrugation torsion stiffness \( K_T \) is determined by equating the external work \( W \) to the internal strain energy \( U \). Section 2.1.3 shows that the most important component of the corrugation torsion strain energy is due to flange out-of-plane bending so that the strain energy due to the flange bending moment, \( UM_{fy} \), should be included in the expression of the internal strain energy. The flanges are treated as wide beams when bent about their weak axes. Other strain energy terms may include the strain energy due to the web vertical shear, \( UV_{wz} \), the strain energy due to the flange vertical shear, \( UV_{zf} \), and the strain energy due to the web in-plane normal force, \( UP_w \).

These strain energy terms can be expressed as

\[
UM_{fy} = \int \frac{\left(1 - \nu^2\right) M_{fy}^2(x)}{2EI_{xf}} \, dx
\]

\[
UV_{wz} = \int \frac{V_{wz}^2(x)}{2GA_w} \, dx
\]

\[
UV_{zf} = \int \frac{V_{zf}^2(x)}{2GA_f} \, dx
\]

\[
UP_w = \int \int \frac{n_z(x, z)^2}{2E \cdot t_w} \, dxdz
\]

where
\[ I_{yf} = \frac{1}{12} b_f t_f^3 \]
\[ A_w = D \cdot t_w \]
\[ A'_f = \frac{5}{6} b_f \cdot t_f \]

Note that the expression of \( U_{P_w} \) consists of only \( n_2(x, z) \) since \( n_1(x, z) \) is zero according to Equations (2.53) and (2.54). Due to the symmetry conditions discussed previously, the strain energy for the flanges within the length of one corrugation is simply eight times of that of the bottom flange between section 6 and section 8. Similarly, the strain energy for the web within the length of one corrugation is four times of that of the web between section 6 and section 8. \( M_{fy}(x), V_{wz}(x), V_{fz}(x) \) have been determined in Section 2.4.2. Substituting them into Equation (2.64) results in

\[
\begin{align*}
UM_{fy} &= 8 \int_0^{b+d} M_{fy}^2(x) dx = \frac{1}{EI_{yf}} \left( B_1 n_1^2 - B_2 c \cdot n_1 \cdot n_i + B_3 \cdot c^2 \cdot n_i^2 \right) \\
UV_{wz} &= 4 \int_0^{b+d} V_{wz}^2(x) dx = \frac{1}{2GA_w} \left( \frac{c}{3} + b \right) c^2 n_i^2 - \frac{2}{3} b^2 c \cdot n_1 \cdot n_i + \frac{2}{15} b^3 n_i^2 \\
UV_{fz} &= 8 \int_0^{b+d} V_{fz}^2(x) dx = \frac{1}{2GA'_f} \left( \frac{b + d}{2} \right) c^2 n_i^2 - \frac{1}{3} b^2 c \cdot n_1 \cdot n_i + \frac{1}{15} b^3 n_i^2
\end{align*}
\]

where
\[
\begin{align*}
B_1 &= \frac{17}{2520} b^5 + \frac{1}{72} b^4 d \\
B_2 &= \frac{1}{30} b^4 + \frac{1}{12} b^3 d + \frac{1}{36} b^2 d^2 \\
B_3 &= \frac{1}{24} b^3 + \frac{1}{8} b^2 d + \frac{1}{12} b \cdot d^2 + \frac{1}{60} d^3
\end{align*}
\]

\( n_2(x, z) \) has been determined for a quarter of a longitudinal web fold and a quarter of an inclined web fold. Due to symmetry, the strain energy within the length of one corrugation is simply eight times larger

\[
\begin{align*}
UP_w &= 8 \int_0^{D/2} \int_0^{b/2} \frac{1}{2E \cdot t_w} \left( -\frac{4 \cdot n_i}{b \cdot D} x \cdot z \right)^2 dx dz + 8 \int_0^{D/2} \int_0^{c/2} \frac{1}{2E \cdot t_w} \left( \frac{2z}{D} n_i \right)^2 dx dz
\end{align*}
\]

where on the inclined fold, the integration is done along the local axis 1 direction. \( UP_w \) turns out to be

\[
UP_w = \frac{b \cdot D}{9E \cdot t_w} n_i^2 + \frac{c \cdot D}{3E \cdot t_w} n_i^2
\]

(2.66)
Now that the components of the strain energy has been expressed in terms of the section forces $n_i$ and $n_j$, they are combined to express the total strain energy $U$ within the length of one corrugation. Different combinations of the components were investigated in different solutions that were developed. For example, when only $UM_{fy}$ is considered, the solution is labeled as S1. The solutions to the corrugation torsion model, based on different strain energy components that were considered are:

Solution S1: $U = UM_{fy}$
Solution S2: $U = UM_{fy} + UV_{wz}$
Solution S3: $U = UM_{fy} + UV_{wz} + UV_{fo}$
Solution S4: $U = UM_{fy} + UV_{wz} + UV_{fo} + UP_w$
Solution S5: $U = UM_{fy} + UV_{fo}$

For each of the five solutions, the total strain energy has the form

$$U = A_1 n_1^2 - A_2 n_1 \cdot n_i + A_3 n_i^2$$

(2.67)

where coefficients $A_1$, $A_2$ and $A_3$ are solution dependent. For example, when solution S4 is considered, $A_1$, $A_2$ and $A_3$ are defined as

$$A_1 = \frac{B_1}{EI_{sf}} + \frac{2b^3}{15GA_w} + \frac{b^3}{15GA_f} + \frac{bD}{9Et_w}$$

$$A_2 = \left( \frac{B_2}{EI_{sf}} + \frac{2b^2}{3GA_w} + \frac{b^2}{3GA_f} \right)c$$

$$A_3 = \left( \frac{B_3}{EI_{sf}} + \frac{3b + c}{3GA_w} + \frac{3b + d}{6GA_f} \right)c^2 + \frac{cD}{3Et_w}$$

Then, substituting $n_i$ and $n_j$ from Equation (2.63), the total strain energy is

$$U = \left( A_1 \lambda^2 - A_2 \lambda + A_3 \right) \frac{T_e^2}{C^2}$$

The external work due to the torque for corrugation torsion $T_e$ within the length of one corrugation is

$$W_e = \frac{1}{2} T_e \phi$$

where $\phi$ is the relative twist over the length of one corrugation. The twist per unit length is

$$\phi' = \frac{\phi}{L_e}$$

Assuming $T_e$ is related to $\phi'$ by

$$T_e = K_{te} \cdot \phi' = K_{te} \frac{\phi}{L_e}$$

then
\[
\phi = \frac{T_c \cdot L_c}{K_{Tc}}
\]

The external work \( W_c \) can be rewritten as

\[
W_c = \frac{L_c \cdot T^2}{2K_{Tc}}
\]

Letting \( W_c = U \), the corrugation torsion stiffness can be defined as

\[
K_{Tc} = \frac{L_c \cdot C^2}{2 \left(A_1\lambda^2 - A_2\lambda + A_3\right)}
\]

As discussed earlier, the corrugation torsion model omitted stresses that correspond to St. Venant torsion of the comparable FWG. Now including St. Venant torsion, the total torsional stiffness for a CWG under uniform torsion, \( K_T \), can be expressed as

\[
K_T = K_{Tc} + K_{Tf}
\]

where \( K_{Tf} \) is the uniform torsion stiffness of a comparable FWG which represents the St. Venant torsion stiffness of the CWG. The total torsional constant for a CWG under uniform torsion, \( J_{cw} \), is defined as

\[
J_{cw} = \frac{K_T}{G}
\]

The total uniform torsion stiffness of a CWG based on the FE analysis is

\[
K_{T,FE} = \frac{T}{\phi'}
\]

where \( T \) is the reaction torque of the CWG determined from the FE analysis results and \( \phi' \) is the twist per unit length. The parameter \( \lambda \) is determined when the following equation is satisfied.

\[
K_{Tc} + K_{Tf} = K_{T,FE}
\]

As suggested in Section 2.3.3, the next step is to select a practical range of CWGs and determine the torsional resistance of all the selected cases based on FE analysis. Then \( \lambda \) is determined for each case and the results are used in a regression analysis for \( \lambda \).

### 2.5 Practical Cases of CWGs for Torsional Analysis

In this section, a range of practical CWGs are selected. These cases have identical top and bottom flanges and the nominal yield stress is assumed to be the same for the flanges and web. The approaches used to select a practical range of CWGs include:

- Using a thin web and a high \( D/t_w \) ratio to take advantage of the high shear buckling resistance of corrugated webs.
- Using compact flanges as is the case for most bridge girders.
- Considering the issue of handling during fabrication and erection.

Referring to existing CWGs and typical conventional steel I-girders for highway bridges, the following dimensions were selected:
\[ b = 300, 450 \text{ mm} \]
\[ c = 250, 300, 375, 450 \text{ mm} \]
\[ \alpha = 30, 36.9 \text{ deg.} \]
\[ b_f = 300, 400, 500, 600 \text{ mm} \]
\[ t_f = 25, 30, 35, 40, 45, 50, 55, 60 \text{ mm} \]
\[ D = 1500, 2000, 2500 \text{ mm} \]
\[ t_w = 6, 9 \text{ mm} \]

A computer program was written to combine these parameters and to select practical cases based on the following requirements

\[
200 \leq \frac{D}{t_w} \leq \min \left( 500, \frac{D}{t_{w, \text{max}}} \right)
\]

\[
0.308 \sqrt{\frac{E}{F_y}} \leq \frac{b_f + c \sin(\alpha)/2}{2t_f} \leq 0.382 \sqrt{\frac{E}{F_y}}
\]

\[
\frac{b_f + c \sin(\alpha)/2}{2t_f} \leq 12
\]

\[
2.5 \leq \frac{D + 2t_f}{b_f} \leq 5
\]

\[
1.0 \leq \frac{b}{c} \leq 1.2
\]

\[ b_f \geq c \sin(\alpha) + 200 \]

where \( \left( \frac{D}{t_w} \right)_{\text{max}} \) is determined from Sause et al. 2003

\[
\left( \frac{D}{t_w} \right)_{\text{max}} = 1.7 \sqrt{\frac{E}{F_{yw}}} \left( \frac{b}{t_w} \right)^{1.5} F(\alpha, \beta)
\]

where

\[
F(\alpha, \beta) = \frac{(1 + \beta) \sin(\alpha)^3}{\beta + \cos(\alpha)} \left[ \frac{3\beta + 1}{\beta^2 (\beta + 1)} \right]^{0.75}
\]

\[ \beta = \frac{b}{c} \]

The first two limits depend on the steel nominal yield stress, which is taken as 485 MPa (70 ksi). 136 cases satisfy these requirements and they are listed in Table 2.5. Properties of these selected cases are
222 ≤ \( \frac{D}{t_w} \) ≤ 417

\[
6.3 \leq \frac{b_f + c \sin(\alpha)/2}{2t_f} \leq 7.7
\]

\[
2.7 \leq \frac{D + 2t_f}{b_f} \leq 4.4
\]

\[
\frac{b}{c} = 1.0 \text{ or } 1.2
\]

\[
213 \leq b_f - c \sin(\alpha) \leq 475
\]

### 2.6 FE Analyses of Selected Cases and Corrugation Torsion Results

The FE model used for the prototype CWG was discussed in Section 2.1.1. Table 2.1 shows that the element size of the model can be changed significantly without significantly changing the reaction torque. Model 62, model 63, and model 64 show that with a relatively fine mesh for the flange, the reaction torque does not change when the number of elements for a web fold is increased from 4 to 8 in the local axis 1 direction. Similarly, model 71, model 72, and model 73 show that with a relatively fine mesh for the web, the reaction torque is nearly the same when the flange mesh is doubled in both the local axis 1 and local axis 2 directions. Similar studies for other CWGs showed similar findings. So the FE mesh (Model 63) selected in Section 2.1.1 for the prototype CWG was used for the FE analyses of all the cases shown in Table 2.5.

The reaction torque, \( T \), for each CWG and the reaction torque, \( T_f \), of the comparable FWG for each of the selected cases were determined and listed in Table 2.5. In Section 2.4, five solutions to the corrugation torsion model were suggested based on different combinations of the strain energy components. First, case NC14 and case NC70 were used to study the different solutions. For this purpose, the torsional stiffness of the comparable FWG is defined to be the result of FE analysis of the comparable FWG.

\[
K_{T_f, FE} = \frac{T_f}{\phi'}
\]

where \( T_f \) is the reaction torque of the FWG determined from the FE analysis results and \( \phi' \) is the twist per unit length. \( \lambda' \) is determined when Equation (2.70) is satisfied. In general, there are two values of \( \lambda' \) which satisfy Equation (2.70), \( \lambda_1 \) and \( \lambda_2 \), where \( \lambda_1 \) is the smaller of the two. To determine which \( \lambda' \) is the correct one, the flange out-of-plane bending moment \( M_f \) and the vertical shear force \( V_f \) are determined using the corrugation torsion model and are compared with the FE analysis results. This comparison is made for each possible solution S1 through S5, described earlier in
terms of the strain energy components included, except for solution S4, for which no value of \( \lambda \) was found that satisfy Equation (2.70). The comparisons of \( M_{fy} \) and \( V_{fz} \) are made for the flange between section 6 and section 8 (see Figure 2.15) and are shown in Figure 2.33 and Figure 2.34. The calculated results compare more favorably with the FE results using \( \lambda \) (Figure 2.33). The differences among the results of the different solutions are small. In particular, the results of solutions S1 and S5 are very close, so are the results of solutions S2 and S3, which indicates the effects of the flange vertical shear \( V_{fz} \) are small. Based on above comparisons, solution S2 with \( \lambda \) was considered to be the best solution. This solution considers the effects of flange out-of-plane bending moment \( M_{fy} \) and web vertical shear force \( V_{wz} \).

In practice, the uniform torsion stiffness of a FWG is calculated from St. Venant torsion theory rather than FE analysis. Figure 2.35 shows a comparison of \( M_{fy} \) and \( V_{fz} \) for cases NC14 and NC70 from the FE analysis results and determined from the solution S2 corrugation torsion model. For the corrugation torsion model, \( \lambda \) was determined using two different calculations. In one, the torsional stiffness of the comparable FWG, \( K_{Tf} \), is determined from the FE analysis, labeled \( K_{Tf\_FE} \). In the other, the torsional stiffness of the FWG is determined from St. Venant torsion theory, labeled \( K_{Tf\_SV} \), which is defined as

\[
K_{Tf\_SV} = GJ \tag{2.72}
\]

where \( J \) is the uniform torsion constant for conventional I-girders defined as

\[
J = \frac{2}{3} b_f t_f^3 + \frac{1}{3} D r_w^3 \tag{2.73}
\]

Figure 2.35 shows that the results from the corrugation torsion model determined using \( K_{Tf\_SV} \) are closer to the FE analysis results, especially near the maximum \( M_{fy} \) of case NC14.

This result was investigated using the data listed in Table 2.6, where \( K_{Tf\_FE} \) indicates when \( K_{Tf} \) is from FE analysis and \( K_{Tf\_SV} \) indicates when \( K_{Tf} \) is from St. Venant torsion theory. \( T_c \) is the reaction torque for corrugation torsion determined as \( T_{FE} - T_{f} \). \( W_e \) is the external work done by \( T_c \), equal to \( T_c \phi / 2 \). \( UM_{fy\_FE} \) is the strain energy from \( M_{fy} \) determined from the FE analysis results, as discussed in Section 2.1.3. \( UM_{fy\_cal} \) is the strain energy from \( M_{fy} \) determined from Equation (2.64).

For case NC14, when \( K_{Tf} \) is from FE analysis, \( W_e \) is 7.87 kN-mm. \( UM_{fy\_FE} \) is 4.57 kN-mm, which is 58% of \( W_e \). \( UM_{fy\_cal} \) is 7.19 kN-mm, which is 91% of \( W_e \). For this case, the calculated \( M_{fy} \) is artificially increased by the assumption that the corrugation torsion resistance is mainly due to flange bending, and the calculated \( M_{fy} \)
is significantly higher than the FE analysis results. When $K_{T_f}$ is from St. Venant torsion theory, $T_f$, equal to 18683 kN-mm, is higher than the value from FE analysis, equal to 17735 kN-mm, which results in a smaller $T_c$. The smaller $T_c$ produces a smaller $W_c$ (5.49 kN-mm), and a small $UM_{fy,cal}$ (5.04 kN-mm), which is much closer to $UM_{fy,FE}$. Therefore, when $K_{T_f}$ is from St. Venant torsion theory, the calculated $M_{fy}$ is much close to the FE analysis results.

For case NC70, when $K_{T_f}$ is from the FE analysis, $UM_{fy,cal}$ is closer to $UM_{fy,FE}$. However, when $K_{T_f}$ is from St. Venant torsion theory, both the calculated $M_{fy}$ and $V_f$ are closer to the FE analysis results (see Figure 2.35).

Based on above analyses, Solution S2 of the corrugation torsion model was used to determine $\lambda$ for all the selected cases. $K_{T_f}$ was determined from Equation (2.72).

The resulting values of $\lambda$ are given in Table 2.5.

### 2.7 Regression Analyses and Evaluation

Regression analyses were performed to develop a function for the parameter $\lambda$ of the corrugation torsion model. In addition, a study of the results in Table 2.5 revealed that a relationship between the total reaction torque $T$ and the reaction torque $T_f$ could be developed. Therefore a regression of the ratio $T/T_f$, where $T$ is determined from the FE analysis and $T_f$ is determined from St. Venant torsion theory was also conducted. $T/T_f$ indicates directly how much the torsional stiffness of a CWG is increased compared to that of the comparable FWG.

The two regressions were performed using the TableCurve 3D program, which has the capacity of doing either $y = f(x)$ type (2D) or $z = f(x, y)$ type (3D) regression. A feature of this program is that it has a large number of built-in functions, and it can select the function based on the $r^2$ value.

There are seven relevant CWG dimensions, which include the corrugation parameters $b, c$, and $\alpha$, the flange width and thickness $b_f$ and $t_f$, and the web depth and thickness $D$ and $t_w$. $\lambda$ and the ratio $T/T_f$ can be functions of any of these dimensions or their combinations. $\lambda$ and $T/T_f$ were plotted against various parameter combinations. Some of these plots are shown in Figure 2.36. The regression parameters used in these figures are
\[
P_1 = \frac{4E_b t_f^3}{(b + c \cos(\alpha))^3}
\]
\[
P_2 = \frac{c^2 \sin(2\alpha)}{2b \cdot b_f}
\]
\[
P_3 = \frac{c \cdot t_w (b + c \cos(\alpha))^3}{2D \cdot b_f t_f^3}
\]
\[
P_4 = \frac{c^2 t_w (b + c \cos(\alpha))^3 \sin(\alpha)}{2D \cdot b_f^2 t_f^3}
\]

(2.74)

It can be seen that when \( \lambda \) or \( T/T_f \) are plotted against either \( P_3 \) or \( P_4 \), the data points are neatly arrayed. Whether \( P_3 \) or \( P_4 \) is the better parameter will depend on the regression results.

Two major branches can be observed on the \( T/T_f \) versus \( P_3 \) (or \( P_4 \)) plot. In Figure 2.37, \( \lambda \) and \( T/T_f \) are plotted against \( P_4 \) for the two different values of \( b/c \) and corrugation angle values used in the study. There are 66 cases with \( b/c = 1.0 \) and 70 cases with \( b/c = 1.2 \). There are 70 cases with \( \alpha = 30 \) and 66 cases with \( \alpha = 36.9 \) deg. Clearly, the separation of the data on the \( T/T_f \) versus \( P_4 \) plot is mainly due to the corrugation angle. For the same corrugation angle, a further separation of the data is due to the different \( b/c \) values. No separation of data is observed for \( \lambda \) versus \( P_4 \).

From above observations, a 2D regression is conducted for \( \lambda \) as a function of either \( P_3 \) or \( P_4 \). Based on the \( r^2 \) value, the better regression turns out to be \( \lambda \) versus \( P_4 \) and the regression equation is

\[
\lambda = 1.329 - 0.7284 \frac{0.009198}{\sqrt{P_4}} - \frac{P_4^2}{P_4^2}
\]

(2.75)

which gives an \( r^2 \) value of 0.94. Figure 2.38(a) shows the data points and the curve from Equation (2.75).

A 3D regression is done for \( T/T_f \) as a function of the corrugation angle \( \alpha \) and either \( P_3 \) or \( P_4 \). The better fit turns out to be \( T/T_f \) versus \( P_4 \) and the regression equation is

\[
\frac{T}{T_f} = \left(0.02233 + \frac{0.05045}{\sqrt{P_4}} + 1.262e^{-\alpha}\right)^{-1}
\]

(2.76)

where the unit of the corrugation angle \( \alpha \) is radians. The regression equation gives an \( r^2 \) value of 0.93. Figure 2.38(b) shows the data points and the corresponding curves from Equation (2.76).

Next, the regression results presented above are evaluated. Equation (2.75) can be used to perform the following calculations:
• The corrugation torsion stiffness, $K_{tc}$, using Equation (2.68).
• The total CWG uniform torsion stiffness $K_T = K_{tf} + K_{tc}$.
• The total CWG uniform torsion constant $J_{cw} = K_T / G$.
• The total CWG uniform torsion reaction torque, $T = K_T \cdot \phi'$.
• The corrugation torsion reaction torque, $T_c = K_{tc} \cdot \phi'$.
• The constant $C$ and the normal section forces $n_l$ and $n_t$ using Equation (2.63).
• The CWG uniform torsion internal forces as formulated in Section 2.4.2.
• The strain energy due to the internal forces as formulated in Section 2.4.3.

Equation (2.76) can be used to perform the following calculations:
• The total CWG uniform torsion stiffness, $K_T = K_{tf} \cdot T / T_f$.
• The CWG uniform torsion reaction torque, $T = T / T_f \cdot K_{tf} \phi'$.
• The total CWG uniform torsion constant, $J_{cw} = K_T / G$.

The evaluation of the regression results was made as follows:
• The total CWG uniform torsion reaction torque $T$ was calculated using the regressions results and was compared to the FE analysis results.
• The internal forces were calculated for selected cases using the regression results for $\lambda$ and compared to the FE analysis results.

As outlined above, the total CWG uniform torsion reaction torque $T$ was calculated, $T_{cal}$, and compared to the FE analysis results, $T_{FE}$, in Figure 2.39 using an error term defined as

$$error = \frac{T_{FE} - T_{cal}}{T_{FE}} \times 100\%$$

It can be seen that the error in the calculated total CWG uniform torsion reaction torque is very small. The error based on the $\lambda$ regression is from -2.1 to 2.6%. The error based on the $T/T_f$ regression is from -3.9 to 2.9%.

In Section 2.6, the bottom flange stress resultants $V_{zf}$ and $M_{fy}$ were compared to the FE analysis results to identify the best solution for the corrugation torsion model. The comparisons were done for two cases, NC14 and NC70. Similar comparisons are made here using $\lambda$ from Equation (2.75). Figure 2.40 shows six calculated stress resultants for the bottom flange together with the FE analysis results for cases NC14 and NC70. The calculated results are from Equation (2.59) and Equation (2.60). Due to symmetry, only the results between section 6 and section 8 (Figure 2.15) are shown. It can be seen that the calculated $M_{fy}$ and $V_{zf}$ are close to the FE results for case NC70, as is the calculated $M_{fy}$ for case NC14. Other calculated internal forces do not compare very well with the FE results.

In summary, the calculated total CWG uniform torsion reaction torques are very close to the FE analysis results. This indicates that the calculated total CWG uniform torsion stiffness $K_T = K_{tc} + K_{tf_{SV}}$ is very close to the stiffness of the FE models. The regression equation for $T/T_f$ provides a convenient way to determine the total CWG
uniform torsion stiffness. Comparisons of the internal forces show that $M_{fy}$ can be predicted very well, which is the most important stress resultant for corrugation torsion according to the kinematic study presented in Section 2.1.4.

2.8 Summary

In this chapter, FE models of the prototype CWG and the comparable FWG were developed. A FE mesh was selected through a mesh convergence test. FE analyses showed that the uniform torsion stiffness of a CWG is larger than that of the comparable FWG. This increased stiffness was investigated through a torsional deformation study and a strain energy study.

Under uniform torsion, the flanges of CWGs are bent out-of-plane and the uniform torsion resistance from this mechanism was named corrugation torsion. Both a kinematic analysis and a simple static analysis based on the approach of Lindner and Aschinger (1990) were used to explain the corrugation torsion mechanism.

Corrugation torsion was a very complicated problem, which was revealed by the internal force distributions from FE analysis. Static equilibrium formulations were developed which showed that corrugation torsion is a highly statically indeterminate problem. Based on the results of FE analysis, web section forces at the web-flange interface were assumed to reduce the degree of indeterminacy, and a corrugation torsion model was proposed.

A static solution was developed by assuming a constant shear flow over the web depth. Internal forces for both the web and the flanges were derived. By equating the external work under uniform torsion attributed to corrugation torsion to the strain energy of the internal forces of the corrugation torsion model, a corrugation torsion stiffness was derived, which depends on an undetermined parameter $\lambda$. A solution for the corrugation torsion model was selected based on the comparisons of the calculated flange internal forces and the FE analysis results.

A large number of CWGs with practical dimensions were selected. The reaction torque under an imposed twist for each CWG and the comparable FWG were determined from FE analyses. The parameter $\lambda$ was then determined for each case. A regression equation was developed for $\lambda$. The results were evaluated by comparing the calculated reaction torque and corresponding internal forces with the FE analysis results. A regression equation was also determined for the ratio of the CWG total reaction torque $T$ to the FWG reaction torque $T_f$, $T/T_f$, which directly indicates how much the torsional stiffness of a CWG is increased relative to that of a comparable FWG.
<table>
<thead>
<tr>
<th>Mesh</th>
<th>Flange</th>
<th>Inclined fold</th>
<th>Longitudinal fold</th>
<th>Elements per corrugation</th>
<th>Reaction torque (kN-mm)</th>
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</thead>
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### Table 2.2 Strain energy of a single corrugation with 50mm flange plate

<table>
<thead>
<tr>
<th>Flanges (kN-mm)</th>
<th>Inclined web fold (kN-mm)</th>
<th>Longitudinal web fold (kN-mm)</th>
<th>Total (kN-mm)</th>
<th>% of $U_{total}$</th>
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</thead>
<tbody>
<tr>
<td>$U_b$</td>
<td>18.12</td>
<td>0.01</td>
<td>18.13</td>
<td>40.7</td>
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<tr>
<td>$U_p$</td>
<td>0.01</td>
<td>0.88</td>
<td>1.03</td>
<td>2.3</td>
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<tr>
<td>$U_{t1}$</td>
<td>23.98</td>
<td>0.19</td>
<td>24.38</td>
<td>54.8</td>
</tr>
<tr>
<td>$U_{t2}$</td>
<td>0.01</td>
<td>0.38</td>
<td>0.96</td>
<td>2.2</td>
</tr>
<tr>
<td>Total</td>
<td>42.13</td>
<td>1.46</td>
<td>44.50</td>
<td>100</td>
</tr>
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<td>% of $U_{total}$</td>
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### Table 2.3 Strain energy of a single corrugation with 40mm flange plate

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<th>Total (kN-mm)</th>
<th>% of $U_{total}$</th>
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### Table 2.4 Strain energy of a single corrugation with 20mm flange plate

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Table 2.6 Comparisons of flange out-of-plane bending strain energy (kN-mm)

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Figure 2.16 Flange section force contour plots on a typical corrugation (continued)
Figure 2.16 Flange section force contour plots on a typical corrugation (continued)
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(continued)
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Figure 2.40 Comparison of internal forces calculated from regression results and FE analysis results (continued)
3 Lateral Torsional Buckling Under Uniform Bending

Though uniform bending is not representative of the loading of actual bridge girders, for conventional steel I-girders, the lateral torsional buckling (LTB) strength under uniform bending serves as the basis for the LTB strength under other loading conditions. The LTB strength under other loading conditions is determined by modifying the LTB strength under uniform bending with a so-called moment gradient factor.

The LTB strength of corrugated web girders (CWGs) under uniform bending (i.e., constant bending moment over the unbraced length) will be discussed in this chapter using the results of numerous finite element (FE) analyses. Both elastic and inelastic LTB strength will be studied. The scope of the study is limited to CWGs with identical top and bottom flanges, and trapezoidally corrugated webs.

3.1 Elastic LTB and Weak Axis Moment of Inertia

The classical expression for the elastic LTB moment for doubly symmetric I-beams under uniform bending moment is assumed initially to be valid for a CWG. This assumption is verified later in this chapter. The classical expression is (Timoshenko and Gere 1963):

\[ M_{cr-e} = \frac{\pi}{L_b} \sqrt{EI_z K_T + \left(\frac{\pi E}{L_b}\right)^2 I_w z_I} \quad (3.1) \]

where \( L_b \) is the lateral unbraced length, \( K_T \) is the uniform torsion stiffness of the CWG, \( E \) is Young’s modulus for steel, equal to 200 kN/mm² (29000 ksi), \( I_w \) is the warping torsion constant, and \( z_I \) is the moment of inertia of the cross section about its weak axis. As discussed in Chapter 2, \( K_T \) should include the effect of the corrugations, through the corrugation torsion resistance under uniform torsion. Assuming the cross section is thin walled, and neglecting any effect of the corrugations on \( I_w \), \( I_w \) equals \( I_f h^2 / 2 \), where \( I_f \) is the moment of inertia of each flange about its strong axis, and \( h \) is the distance between the flange centroids.

Due to the corrugation geometry, the web is generally not in the center plane of the girder (the \( xz \) plane), which could increase the cross-section property, \( z_I \). However, as discussed in Section 1.7, the axial stiffness of the corrugations is assumed to be negligible. So the corrugated web does not carry axial normal stress. Therefore, neglecting the axial stiffness, but not the bending stiffness of the web, \( z_I \) can be determined as

\[ z_I = \frac{1}{6} t_f b_f^3 + \frac{1}{12} D r_w^3 \quad (3.2) \]

or by neglecting entirely the contribution of web, \( z_I = t_f b_f^3 / 6 \).

Finite element (FE) analyses of the LTB of CWGs under uniform bending were performed, as discussed later. Eight cases were selected from among the cases used for
the torsional analyses presented in Chapter 2 as follows. Table 3.1 shows the properties of the selected cases. Cases 53 and 116 are the two cases with the maximum and minimum error in the calculated CWG torsional resistance based on the $\lambda$ regression described in Section 2.7. Cases 93 and 132 are the two cases with the maximum and minimum error in the calculated CWG torsional resistance based on the $T/T_f$ regression described in Section 2.7. Cases 14 and 70 are the two cases that were used to evaluate the regression results (Sections 2.7). Cases 9 and 88 are two randomly selected cases.

To validate the above approach for determining $I_z$ using FE analyses, the eight selected cases were modeled using the ABAQUS v6.3 FE simulation program. Ten corrugations are included in the model for each case, and the model is loaded laterally. Figure 3.1 shows the deformed shape of Case 70. The girder is supported at the web mid-depth nodes at the ends, nodes A and B. At the left end, the displacements of DOF 1 to 5 of node A are restrained, and at the right end, the displacements of DOF 2 to 5 of node B are restrained. The displacements of the other nodes at each end cross section are related to those of the support node by using the “Kinematic Coupling” option in ABAQUS v6.3. At the mid span, a translation in the negative 2-axis direction is imposed on the web center node O. The displacements of DOF 2 of the other nodes on the cross section at the mid span are made equal to that of node O. Using beam theory, the deflection at mid span including the contribution of shear deformation should be

$$
\delta = \frac{PL^3}{48EI} + \frac{f_s P L}{4G A}
$$

where, $P$ is the force at mid span, $L$ is the span length, $f_s$ is a factor related to the cross section shape, with a value of 1.2 for the rectangular cross section of the flanges, $G$ is the shear modulus, equal to 76.9 kN/mm$^2$ (11200 ksi), $A = 2 A_f$, and $A_f$ is the area of one flange. $I_z$ can be estimated from the FE analyses by solving the above equation as follows

$$
I_{zFE} = \frac{P_{FE}L^3}{48E} \left( \delta_{FE} - \frac{3P_{FE}L}{20GA_f} \right)
$$

(3.3)

where $\delta_{FE}$ is the imposed displacement, $P_{FE}$ is the corresponding force, and $I_{zFE}$ is the result. A ratio of $I_{zFE}/I_{z0}$ is shown also in Table 3.1 where $I_{z0}$ is determined from Equation (3.2). It can be seen that $I_{zFE}/I_{z0}$ ranges from 1.001 to 1.01. The difference in $I_z$ is so small that it is concluded that Equation (3.2) can be used to determine $I_z$ for a CWG. Also it is found that the web contribution is so small that it can be dropped from Equation (3.2). $I_{zFE}/I_{z0}$ versus $h_f/b_f$ is plotted in Figure 3.2 and it can be seen that $I_{zFE}/I_{z0}$ increases slightly with an increases in $h_f/b_f$. 

\[115\]
3.2 FE Models for LTB Analysis

FE models for the eight cases were developed using the optimized mesh developed for the torsional analyses described in Chapter 2 with several modifications. First, the 4-node element S4 was used instead of the 8-node element S8R, since nonlinear analyses were found to run much faster with element S4, and no significant difference in the results was observed. Second, end stiffeners are used on the end cross sections, as shown in Figure 3.3. The end stiffener prevents distortion of the end cross section and facilitates the transfer of transverse and vertical shear without local stress concentration. The thickness of the stiffener was determined from Equation (6.10.8.2.2-1) of the AASHTO LRFD Bridge Design Specifications (2000) assuming a full width stiffener. The stiffener is introduced by connecting its nodes to the corresponding nodes on the flanges and web at the end cross section. The uniform torsion reaction torque of the CWG with the end stiffeners was determined from the new FE model for the selected cases and compared to the results from the previous torsional analyses described in Chapter 2, and the difference in the reaction torque was negligible.

The FE models were used for both elastic and inelastic LTB analyses. A series of models were created for each of the selected cases to cover a wide range of lateral unbraced lengths. The smallest model consists of 2 corrugations and the longest model consists of 30 corrugations. For the models in between these limiting cases, an increment of 2 corrugations was used. The FE mesh for the web and flanges is identical for each corrugation except at the ends, where the end stiffeners are included.

The LTB analyses simulate a simply supported CWG braced laterally and torsionally at each end (only) and loaded with a concentrated bending moment, \( M \), acting at each end producing uniform bending moment over the unbraced length. The FE model is supported at the web mid-depth nodes at the two ends, identified as node \( O \) in Figure 3.3. To simulate a simply supported boundary condition, the displacements of DOF 1 to 3 for node \( O \) are restrained at the left end and the displacements of DOF 2 and 3 for node \( O \) are restrained at the right end. To restrain twist of the ends, the transverse displacements of DOF 2 at the centroids of the top and bottom flanges (nodes A and B) are restrained at both ends.

The concentrated moment, \( M \), is applied at node \( O \) in the direction of DOF 5 (see Figure 3.3). Assuming the corrugated web does not carry axial normal stress (see Section 1.7), the applied moment is carried only by the flanges. This is implemented in the FE model by constraining the axial displacement of DOF 1 of the flange nodes (i.e., in the direction of displacement U1 (the 1 direction)) to the displacement and rotation of node \( O \). For node \( A \) at the centroid of the top flange, the constraint is

\[
u_{1_A} = u_{1_O} + \frac{h}{2} u_{5_O}
\]

where \( h \) is the distance between the flange centroids. A similar constraint is used for node \( B \) at the centroid of the bottom flange. The axial displacements of the other nodes on the flanges are related to the displacements of the flange centroids, assuming plane sections remain plane in each flange. For example, for an arbitrary node \( i \) on the top flange (see Figure 3.3),

A linear elastic isotropic material model, defined by a Young’s modulus of 200 GPa (29000 ksi) and Poisson’s ratio of 0.3 was used for the elastic analyses. An isotropic plastic model using von Mises yield criteria with the isotropic hardening and the associated plastic flow rule was used for the inelastic analyses. The von Mises yield criteria is defined by providing a uniaxial stress-strain model. Both the elastic-perfectly-plastic stress-strain model and the realistic stress-strain model based on coupon test data were used, as discussed later.

The shell element available in ABAQUS v6.3 uses a plane stress constitutive assumption. The von Mises yield criterion for a nonlinear hardening material is defined as (Salem 2004)

\[ f = \sigma_e - \sigma_y (\varepsilon_{ps}) \]  

where \( \varepsilon_{ps} \) is the equivalent plastic strain and \( \sigma_y (\varepsilon_{ps}) \) is the yield stress as a function of the equivalent plastic strain, which increases if strain hardening occurs. \( \sigma_e \) is the effective stress, the so-called Mises stress in ABAQUS v6.3, defined as

\[ \sigma_e = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11} \cdot \sigma_{22} + 3\sigma_{12}^2} \]  

The material is elastic when \( f < 0 \) and is yielded when \( f = 0 \).

### 3.3 Elastic Buckling Analyses

Equation (3.1) provides the theoretical elastic LTB strength for a simply supported beam under a uniform moment. For other boundary and loading conditions, the common practice is to modify Equation (3.1) using ideas such as an equivalent lateral unbraced length and a moment gradient factor. Further modifications are also made to account for inelastic buckling, residual stresses, and geometric imperfections. The present study uses the same approach to establish the LTB strength of CWGs. The elastic LTB strength governs the LTB strength of girders with long lateral unbraced lengths. For girders with small or intermediate lateral unbraced lengths, the LTB strength is reduced by inelastic behavior and initial imperfections (including both geometric imperfections such as initial sweep and twist, and material imperfections such as residual stresses). This section describes linear elastic buckling analysis results obtained using the eigenvalue buckling analysis option (Buckle) in the ABAQUS v6.3 FE simulation program.

The elastic buckling analysis results will be presented for the FE models of case NC70. The elastic buckling analyses were conducted with the magnitude of the applied moment equal to the plastic moment \( M_p \) of the CWG. The resulting eigenvalue from the analysis is the ratio of \( M_{cr}/M_p \), where \( M_{cr} \) is the buckling moment from the elastic buckling analysis. \( M_p \) is calculated neglecting the contribution of web as

\[ M_p = A_f \sigma_y h \]
where \( A_f \) is the area of one flange and \( \sigma_y \) is the yield stress of the steel. Figure 3.4 plots the lowest eigenvalue from the eigenvalue analyses (i.e., normalized elastic buckling moment \( M_{cr}/M_p \)) versus the lateral unbraced length for case NC70. The plot is truncated at \( M_{cr} = M_p \). As expected, \( M_{cr} \) decreases with an increase in lateral unbraced length.

The eigenvector associated with the lowest eigenvalue provides the elastic buckled shape. The buckled shapes for case NC70 are shown in Figure 3.5 to Figure 3.7 for different lateral unbraced lengths, given by the number of corrugations in the FE model (i.e., in the unbraced length). The end stiffeners are not shown for clarity in these figures. Figure 3.5 shows the buckled shape of case NC70 with two corrugations. It can be seen that there is no lateral displacement. The top flange (in compression) buckles upward and downward in alternate directions. This buckled shape is a local buckling shape. The maximum displacement is at the center of the longitudinal folds.

Figure 3.7 shows the buckled shape of case NC70 with six corrugations, which includes top flange lateral displacement, and cross section twist. This buckled shape is a LTB shape. The lateral displacement and twist are maximum at the mid span. The same buckled shape is observed for FE models with longer lateral unbraced lengths.

Figure 3.6 shows the buckled shape of case NC70 with four corrugations. This shape is a combination of flange local buckling and LTB. For this particular shape, the maximum displacement is from flange local buckling.

Flange local buckling controls when the unbraced length is short (two corrugations in this case). LTB controls when the unbraced length is long (six corrugations and more in this case). A combination of flange local buckling and LTB is also possible for short unbraced lengths (four corrugations in this case). It is found that these results are typical for the eight cases that were studied.

### 3.4 Nonlinear Finite Element Analyses

Incremental nonlinear inelastic load deflection analyses were conducted using the ABAQUS v6.3 FE simulation program to determine the inelastic LTB strength of CWGs. The modified Riks method available in ABAQUS v6.3 was used for the nonlinear analyses. Geometric imperfections, residual stresses, geometric and material nonlinearity were included as described in the following subsections.

#### 3.4.1 Initial Geometric Imperfection

The elastic and inelastic LTB strength of conventional I-girders is known to depend on the initial geometric imperfection in the girder (Trahair and Bradford 1998). The incremental nonlinear load deflection analyses used here to determine the inelastic LTB strength of CWGs require initial geometric imperfection be added to the FE model geometry. Elastic buckled shapes are often scaled and added to the perfect geometry of an FE models to create an initial geometry with imperfection. This approach is studied using case NC70, with an unbraced length of 14 corrugations (NC70C14). The first three elastic buckling modes for NC70C14 are shown in Figure 3.8. All three modes are LTB mode shapes.
Table 3.2 shows the models investigated to study the imperfection shape for case NC70C14. The elastic buckled shape for model NC70C14 is scaled so the maximum top flange lateral displacement is $L/1000$ and is used as the initial imperfection for model NC70C14EM1, where $L$ is the unbraced length. For model NC70C14COM1, the elastic buckled shape for mode 1 is scaled so the maximum top flange lateral displacement is $L/1000$ and the shape for mode 2 is scaled so the flange displacement is $L/2000$ and the two shapes are combined and used as the initial imperfection. The first three elastic buckled shapes are combined and used as the initial imperfection for model NC70C14COM2, where the first three modes are scaled to produce flange displacements of $L/1000$, $L/2000$ and $L/3000$, respectively. The lateral displacements along the top flange centerline for the three initial imperfection models are shown in Figure 3.9. Comparing the maximum lateral displacement of the three initial imperfections, it can be seen that the largest is from model NC70C14COM2 and the smallest is from model NC70C14EM1.

Nonlinear load deflection analyses were performed on FE models of NC70C14 using the three different imperfection shapes. An elastic-perfectly-plastic stress-strain model was used and residual stresses were not considered. The moment ($M/M_p$) versus left end rotation for the three analyses is plotted in Figure 3.10. The peak values of $M$, $M_{cr}$, are taken as the LTB strength and listed in Table 3.2. It can be noted that even though the maximum values of the compression flange lateral displacement from mode combination 1 and model combination 2 are much larger than that from the elastic buckling mode 1, the moment-rotation curves are nearly the same and so are the peak values of $M$. If the maximum values of the compression flange lateral displacement from the three imperfection patterns are made equal, the imperfection pattern from the elastic buckling mode 1 would be more critical.

The brief study of case NC70C14 showed that the lowest buckling mode shape can be used to provide an effective initial imperfection. However, Figure 3.5 and Figure 3.6 show that the lowest buckling mode may not be a LTB mode when the lateral unbraced length is small. For case NC70, it was observed that the lowest buckling mode is a flange local buckling (FLB) mode for the model with two corrugations (NC70C2) and a combination of FLB and LTB for the model with four corrugations (NC70C4). For the nonlinear LTB study, an initial imperfection shape similar to a LTB shape is needed for models with a small lateral unbraced length to enforce a LTB failure.

ABAQUS v6.3 can also use the deformed geometry from an elastic static analysis as an initial geometric imperfection for a subsequent nonlinear load deflection analysis. Therefore elastic static analyses were used to generate geometric imperfection shapes that resemble LTB mode shapes. In these elastic analyses, a lateral displacement (U2) of 1.0 was applied at the center of the top flange. For the bottom flange, two displacement conditions were investigated. First, when the center of the bottom flange is displaced laterally half as much as the top flange, the imperfection shape is labeled “SD1”. Second, when the bottom flange has no lateral displacement, the imperfection shape is labeled “SD2”. Table 3.3 lists the models investigated to study alternate initial geometric imperfections for nonlinear load
deflection analyses. Models with 6 corrugations (NC70C6) and 14 corrugations (NC70C14) were studied. It can be seen that for the imperfection shape based on the elastic buckling mode 1, labeled “EM1”, the lateral displacement at the center of the bottom flange is between that of imperfection shapes SD1 and SD2.

Figure 3.11 shows the three initial geometric imperfection shapes for NC70C6. It can be seen that imperfection shape SD2 is similar to imperfection shape EM1. The results for NC70C14 are similar and are not shown. These initial geometric imperfections were included in nonlinear load deflection analyses, and the moment versus end rotation plots are shown in Figure 3.12. It can be seen that the moment versus end rotation curves are very close for the three initial imperfection shapes for both NC70C6 and NC70C14. The peak moments are listed in Table 3.3 and it can be seen that the peak moment for imperfection shape SD2 is closer to that for imperfection shape EM1. From these results, it was decided to use the imperfection shape SD2 as the initial geometric imperfection for models with small lateral unbraced lengths as well as models with longer lateral unbraced lengths in the remaining studies. Figure 3.4 shows the normalized LTB strength for case NC70 determined from nonlinear load deflection analyses using imperfection shape SD2, together with results from the elastic buckling analyses. The LTB strength is defined as the peak moment, \( M_{cr} \), from the nonlinear load deflection FE analyses. It can be seen that the LTB strength, \( M_{cr} \), approaches \( M_p \) when the lateral unbraced length decreases and it approaches the elastic LTB strength when the lateral unbraced length increases.

### 3.4.2 Effects of Flange Local Buckling

All the cases listed in Table 3.1 have compact flanges so that the LTB strength is not expected to be affected by flange local buckling (FLB). The compact slenderness limit for CWG is proposed by Sause et al. (2003) as

\[
\lambda_{fp} = 0.382 \sqrt{\frac{E}{F_y}}
\]

where the flange slenderness ratio is defined as

\[
\lambda_b = \frac{b_f + c \sin(\alpha)/2}{2t_f}
\]

Figure 3.13 shows that for a CWG, the unsupported width of the top flange \( b_3 \) is larger than that of a conventional I-girder \( b_1 \). The above flange slenderness ratio is based on \( b_2 \) which is the average of \( b_1 \) and \( b_3 \). This slenderness ratio considers the supporting effects of the two inclined folds.

For a girder with a medium or short unbraced length, the deformed shape from a nonlinear load deflection analysis after the peak moment is reached is usually as shown in Figure 3.14. It can be seen that the top flange deformation is a combination of lateral bending and flange local distortion. The flange local distortion occurs to the left of mid span since at this location there is a large unsupported flange area and the combined compression of the girder primary bending and top flange lateral bending is
large. This deformed shape suggests that the LTB strength may be affected by flange local buckling.

To investigate this possibility, case NC93 is studied since among the selected cases it has the highest slenderness ratio in terms of both $b_2/2t_f$ and $b_3/2t_f$, and it may be more vulnerable to FLB than the other cases. Since the top flange is under greater compression for a girder with a short unbraced length, FLB is more likely. The shortest unbraced length considered in the study of LTB strength is two corrugations (i.e., model NC93C2). If flange local buckling is not a concern for NC93C2, it should not be a concern for girders with longer unbraced lengths. For this study an elastic-perfectly-plastic material is used and effects of residual stress are not considered.

The moment versus left end rotation from the nonlinear load deflection analysis of NC93C2 is plotted in Figure 3.15 where the moment has been normalized by $M_p$. The peak moment (at increment 6 or inc6) and 90% of the peak moment (at increment 19 or inc19) post-peak are identified on the plot. The deformed shapes corresponding to these two points are shown in Figure 3.16 with a scale factor of 10. It can be seen that there is no visible flange plate distortion at the peak moment and a slight lateral bending of the top flange is observed. At 90% of the peak moment post-peak, both the top flange plate bending and top flange lateral bending deformations are clearly visible.

For convenience in discussing these deformations, the directions defined in Figure 3.17 are used. The locations of maximum flange plate bending and flange lateral bending can be determined by checking the bending curvatures. The flange plate bending curvature is defined as

$$\phi_p = \frac{\varepsilon_{US} - \varepsilon_{LS}}{t_f}$$

where $\varepsilon_{US}$ and $\varepsilon_{LS}$ are the axial strain on the upper surface (US) and lower surface (LS) respectively, and $t_f$ is the flange thickness. The flange lateral bending curvature is defined as

$$\phi_l = \frac{\varepsilon_{MS}^N - \varepsilon_{MS}^S}{b_{lp}}$$

where $\varepsilon_{MS}^N$ and $\varepsilon_{MS}^S$ are the axial strain on the middle surface (MS) on the north side (N) and south side (S) of the top flange. $b_{lp}$ is the distance between the location of $\varepsilon_{MS}^N$ and $\varepsilon_{MS}^S$ as shown in Figure 3.18. The axial strains are taken from the element integration points closest to the flange edges, which are shown in Figure 3.18 as small crosses. The calculated curvatures show that at the peak (inc6) the locations of maximum flange plate bending and maximum flange lateral bending coincide and are identified on Figure 3.18 as L1. L2 is a location which is symmetric to L1 about the mid span.

The flange plate bending at L1 is studied in Figure 3.19 where moment is plotted versus the axial strain. The time of peak moment is identified by a small circle in the figure. It can be seen that on the south side (Figure 3.19(a)), the strain on the upper surface is compressive during the analysis while the strain on the lower surface is...
reverses soon after the peak and eventually becomes tensile due to the growth of flange plate bending. On the north side (Figure 3.19(b)), the strains on both surfaces begin to reverse right before the peak due to flange lateral bending. The strain on the lower surface eventually becomes tensile while the strain on the upper surface reverses again, as the effects of flange plate bending exceed the effects of flange lateral bending late in the analysis.

Figure 3.19(c) also shows that the flange plate bending curvature on the south side is much larger than that on the north side near the end of the analysis which is consistent with the deformation shown in Figure 3.16. Figure 3.19(d) shows that flange plate bending on the south side began to grow faster beginning at increment 4 (inc4). The thick dashed line in Figure 3.19(d) shows the flange plate bending curvature under girder primary bending when plane sections are assumed to remain plane. It can be seen that even in the early stages of the analysis, the flange plate bending curvature is larger than that caused by girder primary bending, which indicates that flange plate bending exists from the beginning of loading.

The middle surface axial strains on the south and north side at cross section L1 are shown in Figure 3.20(a). These strains are not influenced directly by flange plate bending. The strain on the south side becomes nonlinear at inc4 and increases faster from inc5. The strain on the north side begins to reverse at inc5 and eventually becomes tensile. The flange lateral bending curvature at this cross section is shown in Figure 3.20(b). It can be seen that flange lateral bending exists from the beginning of loading and grows linearly until inc4.

The above observations suggest that the failure mode shown in Figure 3.16 is due to the combined effects of girder primary bending, top flange lateral bending, and top flange plate bending. The combined effects of girder primary bending and flange lateral bending cause the south side of the top flange to be under greater compression, especially at areas close to the mid span. At the same time, the shaded area shown in Figure 3.21 has a larger unsupported area so that this area is more vulnerable to flange local distortion and local buckling, and flange local distortion is observed in this area as shown in Figure 3.16. It can be seen from Figure 3.19(a) that the strain on the lower surface at the south side did not begin to reverse until inc13 which is well beyond the peak moment at inc6. This means the effects of flange lateral bending are dominant until inc13.

Figure 3.22 shows a schematic plot of the area of yielding of the top flange at cross section L1. For each increment, there are three layers which represent the lower, middle and upper surfaces. On each layer, there are sixteen blocks from left to right which represent the sixteen integration points from the south side to the north side of the flange cross section. It can be seen that at inc5, a significant portion of the top flange has yielded. The yielding reduces the resistance to both flange lateral bending and flange plate bending so that both curvatures grow much faster from inc5 as shown in Figure 3.19(d) and Figure 3.20(b). At the peak (inc6), more than half of the top flange has yielded. After the peak, the flange lateral bending continues to grow and to maintain equilibrium, the applied moment is reduced. The yielded area slowly grows towards the north side of the flange and reaches a maximum at inc11. The growth of flange local distortion at the south side eventually causes the axial strain on the lower
surface to begin to reverse from inc13, causing the stress state on the bottom surface to fall within the yield surface and become elastic seen in the plot for inc14 in Figure 3.22.

Another indication of flange local buckling is the development of both St. Venant torsion and warping torsion stresses on the flange cross section as described by Salem (2004). The von Mises yield criterion (Equation (3.4)) indicates that when the stress state stays on the yield surface, at locations where these torsion-induced shear stresses $S_{12}$ are large and the transverse normal stress $S_{22}$ does not change, the axial normal stress $S_{11}$ is reduced, which may cause the compression flange to carry less compression force and thus cause unloading.

The cross-section identified as L3 in Figure 3.18 is where the largest in-plane shear stress on the top surface was observed, along the south side of the flange. Figure 3.23 shows $S_{12}$ and $S_{11}$ at the south side of flange cross section L3. $S_{12}$ is plotted versus $S_{11}$ together with the von Mises yield surface in Figure 3.23 where $S_{12}$ and $S_{11}$ are normalized by the yield stress, $\sigma_y$. The von Mises yield surface $f = 0$ (Equation (3.4)) is plotted in the $S_{12}$-$S_{11}$ plane by assuming the transverse normal stress $S_{22}$ is zero. The FE analysis results show that all three surfaces at this location yield at inc5 and elastic unloading from the yield surface does not occur. On the upper and lower surfaces, the stress state deviates slightly from the yield surface because of the existence of the transverse normal stress $S_{22}$ at this location. More importantly, the shear stress $S_{12}$ causes a reduction of the axial normal stress $S_{11}$. On the upper surface (Figure 3.23(a)) the shear stress reaches the shear yield stress and the axial normal stress eventually reverses to tension. The normalized axial normal stress and shear stress on the three surfaces are listed in Table 3.4. It can be seen that at inc7, which passed the peak (inc6), $S_{12}$ reached 12% and 11% of shear yield stress $\tau_y$ on the upper and the lower surfaces respectively, but $S_{11}$ is still at 99% and 100% of $\sigma_y$. These results indicate that the shear stress does not cause enough reduction in the axial normal stress to significantly impact the total axial force in the flange.

From the above discussion, it can be concluded that the peak moment for NC93C2 is reached when the girder is subject to the combined action of primary bending and flange lateral bending which grows from the initial imperfection. Some flange plate bending is present, but the peak moment, which represents the LTB strength, is not affected significantly by flange local buckling or flange local distortion.

### 3.4.3 Stress-Strain Models

The preliminary nonlinear load deflection analyses above have used a simple elastic-perfectly-plastic stress-strain model. Since the post yield stress-strain behavior may influence the LTB strength of a CWG, more accurate material models based on coupon test data are developed in this section. Abbas (2003) tested tensile coupons made from HPS 485W high performance steel. The tensile coupons were plate type specimens according to ASTM E8M-00 with a gage length of 200 mm. 22 tensile coupons were tested including 16 coupons from 6 mm thick plate, 2 coupons from 20 mm thick plate and 4 coupons from 50 mm thick plate. It was found that the stress-strain curves from coupons with the same thickness are similar. Stress-strain curves
from coupons with different thickness may be different. Typical stress-strain curves for different thickness coupons are shown in Figure 3.24. The flange thickness for the selected cases listed in Table 3.1 range from 40 to 55 mm and the web thickness is either 6 or 9 mm. Since the stress-strain curve seems mainly affected by plate thickness, the flange material properties for the present study are based on 50 mm coupon tests and the web material properties are based on 6 mm coupon tests.

Figure 3.25(a) shows four stress-strain curves from the 50 mm coupon tests. Using coupon F6-L24 for 50 mm thick plate as an example, the procedures used to construct a stress-strain model from the coupon test data are illustrated. The stress-strain model is the same as that used by Salem (2004) and is shown in Figure 3.26. The stress-strain model is linear up to point A, and after point A the curve begins to soften. Point C marks the 0.2% offset stress (defined as the yield stress) and the corresponding strain. Point B is selected to be at a stress equal to the average of the stress at points A and C. Point D marks the start of strain hardening. Point F marks the ultimate stress. Point E is selected to be at a stress equal to the average of the stress at points D and F. After point F, necking begins and the stress and strain are not uniform over the gage length. The stress-strain behavior after necking is not included in the model.

The transition zone between points A and C is defined by the function
\[ y_a = a \cdot x_a^b \]
where the origin is at point C as shown in Figure 3.26. The stress and strain at points A and B are used to calculate the parameters \( a \) and \( b \)

\[
\begin{align*}
 b &= \frac{\ln\left(\frac{y_A}{y_B}\right)}{\ln\left(\frac{x_A}{x_B}\right)} \\
 a &= \frac{y_A}{x_A^b}
\end{align*}
\]

where
\[
\begin{align*}
 x_A &= \varepsilon_C - \varepsilon_A \\
 y_A &= \sigma_C - \sigma_A
\end{align*}
\]

and
\[
\begin{align*}
 x_B &= \varepsilon_C - \varepsilon_B \\
 y_B &= \sigma_C - \sigma_B
\end{align*}
\]

Similarly, the transition zone between points D and F is defined by the function
\[ y_u = c \cdot x_u^d \]
where the origin is at point F as shown in Figure 3.26. The stress and strain at points D and E are used to calculate \( c \) and \( d \) in a manner similar to \( a \) and \( b \) above.

For coupon F6-L24, the transition curve from A to C is thus defined as
\[ y_a = 8.162 \cdot 10^{37} \cdot x_a^{13.66} \]

and the transition curve from D to F is defined as
\[ y_u = 4.817 \cdot 10^4 \cdot x_u^{2.558} \]

Figure 3.27 shows the stress-strain curves from the coupon test and from the model for F6-L24. It can be seen that the two curves compare very well.
The stress-strain data from the coupon tests and the stress-strain model are engineering stress $\sigma_{\text{eng}}$ and strain $\varepsilon_{\text{eng}}$. The finite elements used in the ABAQUS FE models require the uniaxial stress-strain data in measures of true stress $\sigma_{\text{tr}}$ and true strain $\varepsilon_{\text{tr}}$ (also called natural strain or logarithmic strain). The engineering stress and strain can be converted to true stress and strain using

\[
\begin{align*}
\sigma_{\text{tr}} &= (1 + \varepsilon_{\text{eng}}) \sigma_{\text{eng}} \\
\varepsilon_{\text{tr}} &= \ln(1 + \varepsilon_{\text{eng}})
\end{align*}
\]  

(3.7)

The natural plastic strain is defined as

\[
\varepsilon_{\text{pl}} = \varepsilon_{\text{tr}} - \frac{\sigma_{\text{tr}}}{E}
\]

which together with the true stress defines the plastic behavior of the material. The true stress-strain curve based on the stress-strain model for coupon F6-L24 is also shown in Figure 3.27. It can be seen that the true stress is larger than the engineering stress while the true strain is smaller than the engineering strain. The true stress is increasing even within the so-called “yield plateau”. Since the yield plateau ends at a strain of about 1%, at this point the true stress is only 1% larger than the engineering stress based on Equation (3.7). So for practical purposes, an elastic-perfectly-plastic material can be used if the strain is expected to be small.

The above procedures were performed for all of the coupon tests. The results for the 50 mm coupon tests are listed in Table 3.5. Similar results for the 6mm coupon tests are listed in Table 3.6. The material model used later is based on the average of the coupon tests, so the stresses and strains at the six points in the stress-strain model were averaged, and then the transitions between points A and C and between points D and F were determined based on the averaged values. The data for the average stress-strain model based on the four 50 mm coupons are listed in Table 3.5. Figure 3.28(a) show the average stress-strain model together with the coupon test results for the 50 mm plate. The details from the beginning of softening to the beginning of strain hardening are shown in Figure 3.28(b). It can be seen that the average stress-strain model is a good representation of all the test stress-strain curves.

Since the yield stress of the average stress-strain model is likely different from the nominal yield stress ($\sigma_{\text{yn}}$) of HPS 485W steel, the last step in developing the model for use in the FE analyses is to shift the average stress-strain model so that its yield stress is equal to the nominal yield stress (485 MPa). As illustrated in Figure 3.29, the six characteristic points are shifted to new positions and labeled as $A'$ to $F'$. The stress and strain of point $C'$ are

\[
\begin{align*}
\sigma_{\text{C'}} &= \sigma_{\text{yn}} \\
\varepsilon_{\text{C'}} &= \frac{\sigma_{\text{yn}}}{E} + 0.2\% \\
\end{align*}
\]

where $E$ is the nominal Young’s modulus. For point $A'$,
\[
\begin{align*}
\sigma_{A'} &= \sigma_C \frac{\sigma_A}{\sigma_C} \\
\varepsilon_{A'} &= \frac{\sigma_{A'}}{E}
\end{align*}
\]
where the ratio of \( \sigma_{A'} \) and \( \sigma_C \) is the same as the ratio of \( \sigma_A \) and \( \sigma_C \). For point \( B' \),
\[
\begin{align*}
\sigma_{B'} &= \frac{\sigma_{B'} + \sigma_{C'}}{2} \\
\varepsilon_{B'} &= \varepsilon_C \frac{\varepsilon_{B'}}{\varepsilon_C}
\end{align*}
\]
where the ratio of \( \varepsilon_{B'} \) and \( \varepsilon_C \) is the same as the ratio of \( \varepsilon_B \) and \( \varepsilon_C \). For point \( D' \),
\[
\begin{align*}
\sigma_{D'} &= \sigma_C \\
\varepsilon_{D'} &= \varepsilon_D
\end{align*}
\]
For point \( F' \),
\[
\begin{align*}
\sigma_{F'} &= \sigma_C \frac{\sigma_F}{\sigma_C} \\
\varepsilon_{F'} &= \varepsilon_F
\end{align*}
\]
where the ratio of \( \sigma_{F'} \) and \( \sigma_F \) is the same as the ratio of \( \sigma_C \) and \( \sigma_F \). For point \( E' \),
\[
\begin{align*}
\sigma_{E'} &= \frac{\sigma_{D'} + \sigma_{F'}}{2} \\
\varepsilon_{E'} &= \varepsilon_E
\end{align*}
\]
For points \( D' \) through \( F' \), the strain is kept unchanged. The resulting nominal stress-strain model for the 50 mm plate is shown in Figure 3.30(a). It can be seen that the average stress-strain curve from the coupon tests has been shifted downward to get the nominal stress-strain curve.

The true stress and natural plastic strain from the nominal stress-strain model are used to define the steel material properties of the flange plates in the FE models for the selected cases listed in Table 3.1. The discrete points in the true stress versus natural plastic strain curve used in the FE analyses are listed in Table 3.7.

The nominal stress-strain model for the web plate was obtained similarly based on the 6 mm coupon tests. Figure 3.25(b) shows five stress-strain curves from the 6 mm coupon tests. It can be seen that there is no yield plateau for the 6 mm plate. Since there is no yield plateau, points C and D become one point in the stress-strain model shown in Figure 3.26. The stresses and strains of the characteristic points of the five coupons are identified and listed in Table 3.6 together with the average and nominal stress-strain data. The average and nominal stress-stress models for the 6 mm plate are shown in Figure 3.30(b). It can be seen that for the 6 mm plate, the average stress-strain curve has been shifted upward to get the nominal stress-strain curve. The discrete points in the true stress versus natural plastic strain curve for the web used in the FE analyses are also listed in Table 3.7.
For the end stiffeners of the FE models, an elastic-perfectly-plastic material model with a nominal yield stress of 485 MPa was used.

3.4.4 Residual Stresses Based on Test Data

Residual stresses are introduced during the cooling of a hot-rolled or welded steel member. Flame-cutting will also introduce residual stresses. The residual stresses are introduced by shrinkage of the late-cooling areas of the member, which induces residual compressive stresses in the early-cooling areas. These compressive stresses are equilibrated by tensile stresses in the late cooling areas. Figure 3.31 shows the typical residual stresses in a center-welded hot-rolled plate and hot-rolled plate with flame-cut edges. It is known for conventional I-girders that the LTB strength is decreased due to the presence of residual stresses, especially for I-girders that fail by inelastic LTB, as shown in Figure 3.32. The LTB strength of CWGs will also be reduced by residual stresses. The effects of residual stresses will be investigated in this section. It is assumed that the flange plate is flame-cut, which is typical practice in the fabrication of welded bridge girders. Alpsten and Tall (1970) concluded that the residual stresses in a component plate are a complex superimposed pattern due to:

- Residual stresses from the cooling of the original hot-rolled plate.
- Residual stresses from flame-cutting the plate.
- Residual stresses from welding the plate to other plates.

Alpsten and Tall (1970) show that the distribution of residual stress in heavy plates and shapes is not uniform through the thickness. However, Galambos (1998) found that the calculated strength of columns based on the complete residual stress distribution is only a few percent less than that based on assuming the residual stress is constant through the thickness and equal to the surface-measured value. Based on this information, residual stresses that are constant through the thickness will be used for the study of LTB strength. Tebedge and Tall (1973) concluded that the most important factors that cause variation in residual stresses between different members are the geometry and the fabrication procedure, such as whether the component plate is flame-cut. The effect of yield stress on the residual stress distribution was found to be small. Therefore, the residual stress distribution in a particular cross section may be predicted from data obtained for a similar cross section fabricated using the same procedures.

Experimental data on the residual stresses for conventional I-girders has not been found in the literature. Similarly, experimental data on residual stresses for a CWG has not been found. Experimental data for various H-shapes and component plates intended for columns exists. For a CWG the flange-to-web fillet welds follow the corrugation geometry. Therefore the residual stresses in the flange of a CWG will increasingly differ from those of a comparable FWG as the $h_w/b_f$ ratio of the CWG increases. Among the eight selected cases for LTB study, case NC88 has the largest $h_w/b_f$ ratio and NC88 will be studied initially.

NC88 has flanges made of 500 × 50 mm plate (19.7 × 1.97 inch). Test data on a similar plate (20 × 2 inch) called the “Fritz Plate” by Bjorhovde et al. (1971) are available. The Fritz plate is flame-cut and has two longitudinal centrally located weld beads 38 mm (1.5 inch) apart on the top surface, as illustrated in Figure 3.33. The
distance between the two weld beads is larger than would exist on a CWG with a web
thickness of 9 mm, however the other properties of this plate are applicable to NC88.
The residual stresses on the upper surface, lower surface and the average residual
stresses for the Fritz Plate are shown in Figure 3.34. The two peaks near the middle of
the plate correspond to the two welds. The plate is made from A36 steel (with a
nominal yield stress of 248 MPa (36 ksi)) and it can be seen that the tensile residual
stresses at the flame-cut edges and the welds are actually higher than the yield stress of
the plate material. In these areas, the plate material is subjected to a very steep
gradient of cooling and the tensile residual stresses are equal to the yield stress of the
weld metal or the metal affected by flame-cutting, which is higher than the yield stress
of the unaffected plate material (Brozzetti 1969).

Based on the average residual stresses for the Fritz Plate, the total tension force
on the plate is calculated to be slightly larger than the compression force. A pattern of
modified residual stresses, called the balanced average residual stresses (BAS), were
created by adding a small uniform compression stress to the averaged residual stresses
for the Fritz Plate. Next, the balanced averaged residual stresses were then converted
to rectangular stress blocks which are easier to use in FE models. The balanced
average residual stresses are not perfectly symmetric about the center, so two tensile
stress blocks close to the flange edges were averaged, and the two compressive stress
blocks were also averaged. The results are called the balanced equivalent residual
stresses (BES) and are shown in Figure 3.35 together with the balanced averaged
residual stresses. It can be seen that the magnitude of the balanced equivalent residual
stresses are low compared to the peak values of the balanced averaged residual
stresses. However, the average total tensile force on each edge of the plate, $F_{fc}$
corresponding to flame cutting, and the total tensile force near the middle of the plate,$F_w$ corresponding to welding, are the same for the balanced equivalent residual
stresses and the balanced average residual stresses.

To apply the balanced equivalent residual stresses to the FE model of a flange
plate, the finite element mesh dimensions and the tensile and compressive residual
stress block sizes must be consistent. Figure 3.36 shows that $l_{fc}$ is the size of the
tensile residual stress block at the flange edges and $l_w$ is half the size of the tensile
residual stress block at the middle of the plate. Figure 3.35 suggests that $l_{fc}$ and $l_w$
will directly match the widths of the tensile stress areas observed in experiment, but
this may not be possible in all cases due to FE mesh conditions discussed below for
the FE mesh of a CWG. Therefore, $\sigma_{te}$ , the equivalent tensile residual stresses at the
flange edges, is in general calculated as $F_{fc}/l_{fc} t_f$ and $\sigma_{tc}$ , the equivalent tensile
residual stress at the middle of the plate, is in general calculated as $F_w/2l_w t_f$ . $\sigma_c$
represents the equivalent compressive residual stresses.

There are two methods to apply the balanced equivalent residual stresses. The
first method is to apply the residual stresses directly as initial stress conditions, which
is called “direct input” of the residual stresses. For direct input, $\sigma_{te}$ and $\sigma_{tc}$ are
applied to the corresponding shaded areas and $\sigma_c$ is applied to the un-shaded areas.
The second method applies the residual stresses indirectly. Let $\sigma_{tfc}$ represent the initial tensile stresses caused by flame-cutting, and $\sigma_{tw}$ represent the initial tensile stresses caused by welding. For indirect input, only these initial tensile stresses are applied to the corresponding shaded areas as initial conditions. Then an initial analysis step is used in the FE analysis so that the resulting tensile and compressive stresses will be in equilibrium. By assuming a simple superposition of residual stresses caused by flame-cutting and welding, and assuming the material is elastic, $\sigma_{te}$, $\sigma_{tc}$ and $\sigma_c$ can be expressed as functions of $\sigma_{tfc}$ and $\sigma_{tw}$.

\[
\sigma_{te} = \frac{b_f - 2l_{fc}}{b_f} \sigma_{tfc} - \frac{2l_w}{b_f} \sigma_{tw} \\
\sigma_{tc} = \frac{b_f - 2l_w}{b_f} \sigma_{tw} - \frac{2l_{fc}}{b_f} \sigma_{tfc} \\
\sigma_c = \frac{2l_{fc}}{b_f} \sigma_{tfc} + \frac{2l_w}{b_f} \sigma_{tw}
\]

(3.8)

Since $\sigma_{te}$, $\sigma_{tc}$ and $\sigma_c$ are already known, $\sigma_{tfc}$ and $\sigma_{tw}$ can be determined using either two of these three equations. For the FE mesh shown in Figure 3.36, the initial tensile stresses and the resulting residual stresses for the particular mesh are shown in Figure 3.37. It can be seen that the resulting residual stresses are equivalent to the initial stresses plus a uniform compressive stress over the cross section.

For CWGs, there is no residual stress information available, so the indirect input procedure has to be used. For a CWG, the flange-to-web fillet welds follow the corrugation geometry, so a special FE mesh needs to be developed for which four-node elements (S4) are combined with three-node elements (S3) so that two rows of elements follow the corrugation geometry as shown in Figure 3.38(a). Figure 3.38(b) illustrates the flange mesh adjacent to an inclined fold, where $d$ is the projection of an inclined fold on the longitudinal axis. Two FE meshes are shown in the figure. For the mesh with four rows of elements over the corrugation depth $h_f$, two columns of elements meet at the cross section passing through the center of the inclined fold. For the mesh with three rows of elements over the corrugation depth, the cross section passing the inclined fold center cuts through a single column of elements. Since the FE mesh ends at the centers of the inclined folds at the ends of the model, an even number rows of elements should be used over the corrugation depth. Due to this FE mesh condition, the width of the elements that represent the tensile stress areas in the FE mesh will differ from the width of the tensile stress areas shown in Figure 3.35. As noted above, the tensile forces $F_{fc}$ and $F_w$ are kept constant and $\sigma_{te}$ and $\sigma_{tc}$ are calculated as $\sigma_{te} = F_{fc}/l_{fc} t_f$ and $\sigma_{tc} = F_w/2l_w t_f$. The shaded areas shown in Figure 3.38(c) are where the initial tensile stresses are applied.

For the FE mesh shown in Figure 3.38(c), $\sigma_{tfc}$ is applied to the shaded areas near the flange edges and $\sigma_{tw}$ is applied to the shaded area adjacent to the longitudinal fold of the web, as illustrated in Figure 3.38(d). Since the width of the
shaded area adjacent to the inclined fold is smaller that that adjacent to the longitudinal fold, the initial tensile stress adjacent to the inclined fold, $\sigma_{n0i}$, is calculated assuming that the tensile force $F_{w0}$ is constant as follows

$$F_{w0} = 2\sigma_{n0i}l_{wi}t_f = 2\sigma_{n00}l_{w}t_f$$

where $l_{wi}$ and $l_w$ are as shown in Figure 3.38(c). Thus, the initial tensile stress adjacent to the inclined fold is

$$\sigma_{n0i} = \frac{l_w}{l_{wi}}\sigma_{n0} = \frac{\sigma_{n00}}{\cos(\alpha)}$$

where $\alpha$ is the corrugation angle. The stress $\sigma_{n0i}$ has to be transformed into stresses in local directions 1 and 2, shown in Figure 3.38(d), to be input to ABAQUS v6.3. For an element in area A in Figure 3.38(d), the transformation formulas are

$$\sigma_{11} = 0.5\sigma_{n0i}\cos(2\alpha)$$
$$\sigma_{22} = 0.5\sigma_{n0i}\sin(2\alpha)$$
$$\sigma_{12} = -0.5\sigma_{n0i}\cos(2\alpha)$$

For an element in area B in Figure 3.38(d), the transformation formulas are

$$\sigma_{11} = 0.5\sigma_{n0i}\cos(2\alpha)$$
$$\sigma_{22} = 0.5\sigma_{n0i}\sin(2\alpha)$$
$$\sigma_{12} = 0.5\sigma_{n0i}\cos(2\alpha)$$

These initial stresses are applied to the corresponding areas as initial conditions, and an initial analysis step is used so that the resulting residual stresses will be in equilibrium.

Figure 3.39 shows the deformed shape (enlarged) after the residual stresses are in equilibrium. The girder is shortened which produces compressive stresses to balance the initial axial tensile stresses, and the flanges bend in-plane to balance the off-center tensile stresses along the corrugations. FE analysis results show that the resulting stresses are rather complicated. They are not constant and repeat from one corrugation to another. The residual stresses are small in the web except at locations close to the flanges. The resulting residual stresses in the longitudinal direction, $S_{11}$, for the area of the bottom flange between the centers of two adjacent longitudinal folds are shown in Figure 3.40. It can be seen that the $S_{11}$ pattern has a 2-fold rotational symmetry about the center of the inclined fold. $S_{11}$ is not constant across the flange and an alternating pattern of in-plane bending balances the off-center initial tensile stresses due to welding. The magnitude of the residual stress is largest near the flange edge farthest from the longitudinal web fold since the resulting moment, which balances the off-center initial tensile stresses, produces tension near this edge. The initial tensile stresses and the resulting residual stresses at the center of a longitudinal fold (BF1, identified in Figure 3.40) are shown in Figure 3.41(a). It can be seen that the resulting residual stresses are not uniform across the flange width. The axial residual stresses, $S_{11}$, at the centers of two adjacent longitudinal folds (BF1 and BF2 identified in Figure 3.40) are shown in Figure 3.41(b). It can be seen that the residual stresses at BF1 are symmetric with respect to the flange centerline to the residual
stresses at BF2. Along cross sections between the centers of adjacent longitudinal folds, the residual stresses vary between the two sets of residual stresses shown in Figure 3.41. The residual stress pattern shown in Figure 3.40 and Figure 3.41 is named “RS1”.

The residual stresses for the flange plate with a straight weld has a low compressive stress 47.2 MPa (6.8 ksi) as shown previously in Figure 3.37. This compressive stress is much smaller that the maximum compressive residual stress specified in the AISC LRFD Specifications (1994) for the flange of a welded built-up steel member, which is 114 MPa (16.5 ksi). So another set of residual stresses are considered, where the initial tensile stresses are scaled so that the resulting compressive residual stress will be equal to 114 MPa in the flange plate with a straight weld, assuming elastic material properties. This second set of residual stresses is named “RS4”.

It should be mentioned that residual stresses RS4 may overestimate the magnitude of the residual stresses since 114 MPa (16.5 ksi) is specified as the maximum compressive residual stress of a welded shape. The average compressive stress should be smaller.

### 3.4.5 Parametric Studies of Case NC88

The effects of imperfection amplitude, stress-strain model and residual stresses are studied in this section for case NC88 using the FE mesh developed for the residual stress study. Three imperfection amplitudes are considered: L/1500, L/1000 and L/750. Two steel stress-strain models are considered. The first model is the nominal stress-strain model (Figure 3.30), which is called the “Realistic” material model. The second model is the elastic-perfectly-plastic stress-strain model, which is called the “EPP” material model. The effects of residual stress are studied by using the RS1 and RS4 residual stress patterns. Cases without residual stress are named RS0. The cases considered in this parametric study are illustrated in Figure 3.42. The base case is defined as a model with the realistic stress-strain model, an initial imperfection with an amplitude of L/1000, and residual stresses RS4.

The effects of the stress-strain behavior are studied by changing the stress-strain model to elastic-perfectly-plastic (EPP). The effects of initial imperfection amplitude are studied by changing the imperfection amplitude of the base case to L/1500 or L/750. The effects of residual stresses are studied by changing the residual stress pattern of the base case to RS1 or RS0.

For each of the cases shown in Figure 3.42, a group of FE models with different lateral unbraced lengths are studied. For NC88, the group of FE models includes models with 2, 4, 6, 8, 10, 12, 14 and 16 corrugations. This group of models covers a range of unbraced lengths to make the transition from the plastic moment region to the elastic LTB region.

Nonlinear load deflection analysis results for the cases that illustrate the effects of initial geometric imperfection amplitude are shown in Figure 3.43. It can be seen that the larger the initial geometric imperfection, the smaller the LTB strength. The difference in LTB strength caused by the three imperfection amplitudes is not large.
Analysis results that illustrate the effects of the steel stress-strain model are shown in Figure 3.44. It can be seen that there is no significant difference except when the unbraced length is very short, where using a realistic stress-strain curve gives a slightly higher ultimate strength. Figure 3.45 compares the Realistic and EPP stress-strain curves and it can be seen that there is not much difference between the two curves until strain hardening begins. The small effect of the stress-strain curve indicates that the strains are still small and the strain hardening region was not reached before the peak moment (LTB strength) was reached.

Analysis results that illustrate the effects of residual stresses are shown in Figure 3.46. It can be seen that the LTB strength is reduced due to the presence of residual stresses. Also the larger the residual stress amplitude, the smaller the LTB strength. Residual stress pattern RS1 did not cause significant strength reduction except for models with four and six corrugations. Residual stress pattern RS4 reduced the strength for all the lateral unbraced lengths studied, especially for models with four and six corrugations.

It can be concluded from the parametric study of case NC88 that the LTB strength decreases with increases in the initial geometric imperfection amplitude and residual stresses. For this particular case, the LTB strength is not affected by the stress-strain model.

### 3.4.6 Residual Stresses Based on Formulas

For the seven selected cases other than NC88 listed in Table 3.1, experimental residual stress data for a corresponding flange plate does not exist. Since plate geometry is a major factor that affects the magnitude and distribution of residual stresses, it is not appropriate to simply apply the residual stresses of case NC88 to the other cases. This section develops a consistent method for generating residual stresses for the selected cases using the method from ECCS (1976), which applies to flame-cut, center-welded plates. The ECCS formulas combine the residual stresses due to flame-cutting and welding, and produce self-equilibrating stresses. Figure 3.47 shows the ECCS residual stress model for a flange plate. The ECCS formula for width of the tensile stress block due to flame cutting is

\[
l_{f,c,E} = \frac{1100 \sqrt{f}}{\sigma_y} \text{ (mm)}
\]

where \(\sigma_y\) is the flange yield stress in MPa. For the selected cases, 8 mm web-to-flange fillet welds are assumed. In addition, to maximize the residual stresses, both fillet welds (on each side of the web) are assumed to be deposited simultaneously. The total shrinkage force due to welding is (Young, et al. 1973 and Young 1974)

\[
F_w = pC_0A_{weld}
\]

where \(p\) is welding process efficiency factor as listed in Table 3.8 (Young 1974) and \(C_0\) is a constant, equal to 12000 MPa. The ECCS formula for one half of the width of the area of the tensile residual stresses due to welding is

\[
l_{w,E} = \frac{F_w}{\sigma_y \sum t} \text{ (mm)}
\]
Assuming that the web-to-flange fillet welds are made by submerged arc welding, and consistent with the assumptions stated in Section 1.7, the corrugated web does not carry any weld shrinkage force, the half width of the tensile stress area is

\[ l_{w,E} = \frac{5.4 A_{weld}}{\sigma_y l_f} \text{ (mm)} \] (3.10)

The compressive residual stresses are determined from force equilibrium with the tensile residual stresses remaining at yield. For example, the compressive residual stress magnitude corresponding to flame cutting is

\[ \sigma_{cf_c} = \frac{\sigma_y 2l_{f_{c,E}}}{b_f - 2l_{f_{c,E}}} \]

The total residual stresses are determined by superposing the residual stresses due to flame cutting with those due to welding, as illustrated in Figure 3.47(c). These stresses are named ECCS residual stresses for the convenience of discussion.

In applying Equation (3.10) to the selected study cases, a calculation error was made which increased the value of \( l_{w,E} \) by about 10%. Since the stress in this area is assumed to be \( \sigma_y \), this error introduced approximately 10% more residual stress due to the fillet weld into the flanges.

The residual stress pattern for a flange plate identical to that of NC88 based on the ECCS formulas are shown in Figure 3.48(a), and is identified as SW0. The effect of changing the width of the tensile stress areas was investigated since, as noted previously for \( l_{fc} \) and \( l_w \), the values of \( l_{f_{c,E}} \) and \( l_{w,E} \) may not correspond directly to the FE element sizes due to the FE mesh condition. For the purpose of this analysis, a conventional I-girder corresponding to case NC88 was used. Only the flanges are assumed to be subjected to residual stresses. On either flange, the residual stresses are self-equilibrating. To prevent web local buckling, cross section distortion is prevented. The realistic stress-strain model and an initial imperfection with an amplitude of L/1000 are used for these analyses. The lengths of the I-girder FE models are equal to the lengths of corresponding CWGs with 2, 4, 6, 8, 10, and 12 corrugations.

Studies of the width of the tensile stress areas were carried out as follows. First, let \( l_{f_{c,FE}} \) and \( l_{w,FE} \) represent the width of tensile residual stress areas due to flame cutting and welding in the FE models. For the first study, \( l_{w,FE} \) was set equal to \( 2l_{w,E} \) but the total tension force from welding was unchanged. \( l_{f_{c,FE}} \) was set equal to \( l_{f_{c,E}} \). The resulting residual stress pattern is named SW1 and is also shown in Figure 3.48(a). It can be seen that the magnitude of the tensile stress in the middle of the flange plate is reduced by half. The width of the compressive stress area is reduced and the magnitude of compressive residual stress is slightly increased. Nonlinear load deflection analyses were performed for the FE models with residual stress pattern SW0 and models with residual stress pattern SW1 and the normalized LTB strengths are shown in Figure 3.48(b). It can be seen that the two residual stress patterns, SW0 and SW1, result in the same LTB strength.
For the second study, $l_{fc\_FE}$ was set equal to $2l_{fc\_E}$ but the total tension force from flame cutting was unchanged. $l_{w\_FE}$ was set equal to $2l_{w\_E}$ but the total tension force from welding was unchanged. The resulting residual stress pattern is named SW2 and is shown in Figure 3.49(a). It can be seen that the magnitude of the tensile stress at flange edges is reduced by half. The width of the compressive stress area is reduced and the magnitude of compressive residual stress is slightly increased. Nonlinear load deflection analyses were performed for FE models with residual stress pattern SW2. The normalized LTB strengths are compared in Figure 3.49(b) with results from residual stress pattern SW1. It can be seen that LTB strengths for SW1 are generally lower than those for SW2 even though the magnitude of compressive residual stress of SW2 is slightly higher.

The results of these studies show that increasing the width of either tension stress area decreases the width of the compressive stress areas. The magnitude of the compressive residual stress increases slightly as the width decreases. Increasing $l_{fc\_FE}$ has a larger effect on the LTB strength because it moves the area of residual compressive stress farther from the flange edges. During the nonlinear load deflection analysis, the top flange is under the combined effects of girder primary bending and flange lateral bending. When the compressive stress areas are closer to the flange edges, yielding occurs earlier, reducing the LTB strength. Increasing $l_{w\_FE}$, however, does not have a significant effect. Therefore, for the FE meshes used in the remaining studies, $l_{fc\_FE}$ was kept equal to $l_{fc\_E}$, but $l_{w\_FE}$ was increased from $l_{w\_E}$ when needed to satisfy FE model meshing conditions.

The LTB strength of NC88 with the residual stresses based on the ECCS formulas are compared with those with residual stress pattern RS4 in Figure 3.50. It can be seen that the residual stresses based on the ECCS formulas result in a higher LTB strength for intermediate lateral unbraced lengths. Considering the conservativeness of the residual stress pattern RS4, the results using the ECCS residual stress pattern are considered reasonable.

### 3.5 Lateral Torsional Buckling Strength under Uniform Bending Moment

#### 3.5.1 Analyses of Lateral Torsional Buckling Strength

For the eight selected cases listed in Table 3.1, finite element meshes for the flanges were developed considering the ECCS residual stress pattern. First $l_{fc\_E}$ and $l_{w\_E}$ were calculated and are given in Table 3.9. Elements with different widths were used across the width of the flange, as illustrated in Figure 3.51(a). Two elements, each $l_{fc\_E}$ wide, are used at each flange edge. As discussed previously, an even number of elements is required over the corrugation depth $h_c$ and the width of the elements in this area $l_{w\_FE}$ was selected to be between $l_{w\_E}$ and $2l_{w\_E}$. The number of elements over the corrugation depth area is defined as N1. Based on these two criteria,
the eight cases are divided into three groups. Group A includes cases NC9 and NC14 and has N1 equal to 6. Group B includes cases NC53, NC70, NC116 and NC132 and has N1 equal to 8. Group C includes cases NC88 and NC93 and has N1 equal to 10. Typical flange FE meshes over one corrugation for Group A, B and C are shown in Figure 3.51(b)-(d). Element widths \( l_{w,\text{FE}} \) and N1 are also given in Table 3.9. As discussed earlier, these meshes use both element S4 and element S3. Also, these FE meshes are much finer than those used by Ibrahim (2001), as discussed in Chapter 1.

The element width over the corrugation depth \( l_{w,\text{FE}} \) was calculated by dividing \( h_r \) by the number of elements N1-2. The number of elements N2 between the elements at the flange edge and the elements over the corrugation depth area, was selected loosely based on the criteria that element width in this area, \( e_{cl} \), is between \( \sigma_{fc} \) and 2 \( \sigma_{fc} \). N2 and \( e_{cl} \) are also given in Table 3.9.

The initial tensile stresses are listed in Table 3.10 where \( \sigma_{fc0} \) is the initial tensile stress corresponding to flame cutting, \( \sigma_{w0} \) is the initial tensile stress corresponding to welding adjacent to the longitudinal fold, and \( \sigma_{110} \), \( \sigma_{220} \) and \( \sigma_{120} \) are the components of \( \sigma_{w0i} \), the initial tensile stress corresponding to welding adjacent to the inclined fold, in the local directions, as illustrated in Figure 3.38(d).

With the FE meshes described above, the LTB strength for each of the selected cases listed in Table 3.1 was determined by nonlinear load deflection analysis of FE models with different unbraced lengths. The FE models use the realistic stress-strain model, an initial imperfection with an amplitude of L/1000, and the ECCS residual stress pattern.

Figure 3.52 shows the moment versus left end rotation of case NC88 for the six unbraced lengths studied, where the unbraced length is given by the number of corrugations. The peak moment, \( M_{cr} \), is identified by a small circle in each case. It can be seen that for model with six corrugations (C6) the curve has essentially no softening before the peak, while softening is more obvious when the span becomes longer (C8, C10 and C12) or shorter (C4 and C2).

For NC88, Figure 3.53 shows the moment versus the lateral displacement of the center of the top flange, normalized by L/1000. It can be seen that the longer the unbraced length, the larger the lateral displacement at the peak moment. For shorter unbraced lengths, the behavior is close to in-plane bending behavior and for longer unbraced length, the behavior is more dominated by out-of-plane behavior or lateral torsional behavior. So the softening observed in Figure 3.53 for short unbraced lengths is mainly due to yielding, while for long unbraced lengths, it is mainly due to lateral displacement and twist.

LTB strength is usually presented in the format of peak moment versus lateral unbraced length plot. The LTB strengths from the nonlinear load deflection analyses, \( M_{cr} \), for the eight selected cases are shown in Figure 3.54 to Figure 3.61 together with the elastic LTB strength. In these figures, the LTB strength is normalized by the
plastic moment \( M_p \). Also shown in the figures are the nominal LTB capacity, \( M_n \), based on the following formula from the rules of DIN 18800 part 2 (Lindner 1990)

\[
\frac{M_n}{M_p} = \left( \frac{1}{1 + \lambda_M^{2\alpha}} \right)^{1/n}
\]

(3.11)

In DIN 18800 part 2, \( n = 2 \) for welded beams (Lindner 1990), \( \lambda_M \) is a relative slenderness ratio for bending which is defined as

\[
\lambda_M = \sqrt[2n]{\frac{M_p}{M_{cr,e}}}
\]

where \( M_{cr,e} \) is the elastic LTB moment from Equation (3.1) with \( K_T \) based on the \( \lambda \) regression results, described in Section 2.7. It can be seen that Equation (3.11) with \( n = 2 \) agrees well with the LTB strength from the nonlinear load deflection analyses for CWGs with long lateral unbraced lengths. However, Equation (3.11) overestimates the LTB strength of CWGs with small lateral unbraced lengths (roughly 10 m and less). The value of \( n \) can be reduced to make Equation (3.11) a lower bound to the results from the nonlinear analyses. For example, for case NC9, \( n = 1.3 \) produces a lower bound, but the resulting formula is overly conservative when the lateral unbraced length is between roughly 8 and 16 meters as shown in Figure 3.54.

### 3.5.2 Proposed LTB Strength Formula for Uniform Bending

To have the nominal LTB capacity, \( M_n \), better approximate the nonlinear load deflection analysis results from the FE models over the full range of lateral unbraced length, Equation (3.11) is modified as follows

\[
\frac{M_n}{M_p} = \frac{1}{\sqrt{1 + \lambda_M^{2\alpha}}}
\]

(3.12)

where

\[
\alpha = \begin{cases} 
1 & \text{when } \lambda_M \leq 1 \\
2 & \text{when } \lambda_M > 1
\end{cases}
\]

Equation (3.12) is the same as Equation (3.11) when \( \lambda_M > 1 \). At \( \lambda_M = 1 \), which corresponds to \( M_n/M_p = 0.7 \), a transition is made to a new formula with \( \lambda_M \) raised to the power 2. Equation (3.12) is also shown in Figure 3.54 to Figure 3.61 as “Proposed formula”. It can be seen that Equation (3.12) agrees favorably with the LTB strength from the nonlinear analyses, \( M_{cr} \), over the full range of lateral unbraced lengths and is proposed as the formula to calculate the LTB strength of CWGs under uniform bending.

Section 2.7 shows that the uniform torsion constant of CWGs can be determined based on either the \( \lambda \) regression results or the \( T/T_f \) regression results. Figure 3.62 compares the LTB capacity of the eight selected cases using Equation (3.12) with the uniform torsion constant for CWGs based on the \( \lambda \) regression results (\( J_{cw} \)), the \( T/T_f \)
regression results ($J_{cw}$), as well as with the uniform (St. Venant) torsion constant for conventional I-girders ($J$). It can be seen that the LTB capacity is the same with either $J_{cw1}$ or $J_{cw2}$. Thus, the results show that for the purpose of determining the LTB capacity of CWGs, either $J_{cw1}$ or $J_{cw2}$ can be used. Using $J$ results in smaller LTB capacities for CWGs with long unbraced lengths, but the reduction is generally small, especially for cases NC9 and NC14.

The nominal LTB capacities of the eight selected cases were also calculated using the AASHTO LRFD Bridge Design Specifications Appendix A (2004). As discussed in Section 1.7, the corrugated web does not participate in resisting primary bending, so the restrictions in the AASHTO LRFD Bridge Design Specifications regarding web slenderness are relaxed. Two uniform torsion constants are used with the AASHTO formulas. One is the uniform torsion constant $J$ for conventional I-girders as defined in the specifications

$$J = \frac{1}{3} Dr^3 + \frac{2}{3} b f^3 \left(1 - 0.63 \frac{f_t}{b f} \right)$$

de the other is the uniform torsion constant for CWGs based on the $\lambda$ regression results, labeled $J_{cw}$ thereafter. The torsional constants will affect two variables, $L_r$ and $F_{cr}$, where $L_r$ is the lateral unbraced length at which the transition from elastic to inelastic LTB occurs and $F_{cr}$ is the elastic LTB stress. Since $J_{cw}$ considers the effects of corrugation torsion, it is larger than $J$ for conventional I-girders, which results in a larger $L_r$ and $F_{cr}$.

The nominal AASHTO LTB capacities are compared with those from Equation (3.12) in Figure 3.63. The results from Equation (3.12) are identified as “Proposed formula”. It can be seen that $J_{cw}$ does not cause any significant difference in capacity for case NC9 and NC14. For the other cases, $J_{cw}$ results in a larger elastic LTB strength. Except for case NC132, $J_{cw}$ does cause a significant difference in $L_r$.

It can be seen from Figure 3.63 that the nominal LTB capacities from the AASHTO LRFD Bridge Design Specifications are unconservative for CWGs with intermediate or short lateral unbraced lengths. Using $J_{cw}$, the nominal AASHTO LTB capacities approach the results from the proposed formula (Equation (3.12)) when the lateral unbraced length is long.

### 3.6 Summary

In this chapter the LTB strength of CWGs under uniform bending moment is studied and a formula for the nominal LTB capacity is developed. Both elastic and inelastic LTB strengths, determined by FE analyses, were studied. The elastic LTB strength was determined by eigenvalue elastic buckling analysis (using the Buckle command in ABAQUS v6.3). The inelastic LTB strength was determined from an incremental nonlinear load deflection analysis (using the modified Riks method available in ABAQUS v6.3). The nonlinear analyses included material and geometric non-linearity.
For the nonlinear analyses, great care was taken to incorporate the most important factors affecting the LTB strength into the FE models. These factors include initial geometric imperfections, realistic steel stress-strain models, and residual stresses. The effects of these factors were investigated and the results show that the LTB strength of the cases studied is not significantly affected by the stress-strain model, but the initial geometric imperfection magnitude is important. The LTB strength of CWGs with intermediate lateral unbraced lengths is also significantly affected by residual stresses.

The LTB strengths of eight selected cases were determined from nonlinear load deflection analysis of the FE models and the results were compared with the nominal LTB capacities from both the AASHTO LRFD Bridge Design Specifications (2004) and the German DIN 18800 specifications. Good agreement between the FE results and the nominal LTB capacities from these specifications was not observed, so a new LTB capacity formula was developed by modifying the formula from the DIN specifications. Good agreement was observed between the FE results and the proposed nominal LTB capacity formula.
### Table 3.1 Selected cases for LTB analysis

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<th>$a$</th>
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<th>$t_f$</th>
<th>$D$</th>
<th>$t_w$</th>
<th>$D/t_w$</th>
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<th>$d/b_f$</th>
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<td>4.3</td>
<td>0.21</td>
<td>1.001</td>
</tr>
<tr>
<td>NC88</td>
<td>450</td>
<td>450</td>
<td>36.9</td>
<td>500</td>
<td>50</td>
<td>2000</td>
<td>9</td>
<td>222</td>
<td>6.4</td>
<td>4.2</td>
<td>0.54</td>
<td>1.01</td>
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</tbody>
</table>

*$\lambda_f$ defined by Equation (3.6).
Table 3.2 Comparisons of initial geometric imperfections for NC70C14

<table>
<thead>
<tr>
<th>Imperfection</th>
<th>Imperfection Magnitude</th>
<th>$M_{cr}/M_p$</th>
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</thead>
<tbody>
<tr>
<td>EM1</td>
<td>$L/1000 \times Model$</td>
<td>0.2405</td>
</tr>
<tr>
<td>COM1</td>
<td>$L/1000 \times (Model + Mode2/2)$</td>
<td>0.2404</td>
</tr>
<tr>
<td>COM2</td>
<td>$L/1000 \times (Model + Mode2/2 + Mode3/3)$</td>
<td>0.2407</td>
</tr>
</tbody>
</table>

Table 3.3 Alternate initial imperfections

<table>
<thead>
<tr>
<th>Model</th>
<th>Imperfection</th>
<th>Top Flange Center U2</th>
<th>Bottom Flange Center U2</th>
<th>$M_{cr}/M_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC70C6</td>
<td>EM1</td>
<td>1.0</td>
<td>0.06</td>
<td>0.7164</td>
</tr>
<tr>
<td></td>
<td>SD1</td>
<td>1.0</td>
<td>0.5</td>
<td>0.7223</td>
</tr>
<tr>
<td></td>
<td>SD2</td>
<td>1.0</td>
<td>0.0</td>
<td>0.7184</td>
</tr>
<tr>
<td>NC70C14</td>
<td>EM1</td>
<td>1.0</td>
<td>0.23</td>
<td>0.2405</td>
</tr>
<tr>
<td></td>
<td>SD1</td>
<td>1.0</td>
<td>0.5</td>
<td>0.2412</td>
</tr>
<tr>
<td></td>
<td>SD2</td>
<td>1.0</td>
<td>0.0</td>
<td>0.2402</td>
</tr>
</tbody>
</table>

Table 3.4 Normalized axial normal stress and shear stress (%)

<table>
<thead>
<tr>
<th>Increment</th>
<th>Lower surface</th>
<th>Middle surface</th>
<th>Upper surface</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{12}/\sigma_y$</td>
<td>$\sigma_{11}/\sigma_y$</td>
<td>$\sigma_{12}/\tau_y$</td>
</tr>
<tr>
<td></td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>4</td>
<td>-1.9</td>
<td>-81</td>
<td>-0.5</td>
</tr>
<tr>
<td>5</td>
<td>-5.0</td>
<td>-100</td>
<td>-0.1</td>
</tr>
<tr>
<td>6 (peak)</td>
<td>-8.4</td>
<td>-100</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>-11</td>
<td>-100</td>
<td>0.0</td>
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</table>
Table 3.5 Stress-strain model data for 50 mm coupons

<table>
<thead>
<tr>
<th>Coupon</th>
<th>E (MPa)</th>
<th>A* (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>B* (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>C** (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>D** (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>E** (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>F** (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F5-T20</td>
<td>200000</td>
<td>503.5</td>
<td>0.004518</td>
<td>503.5</td>
<td>0.01270</td>
<td>544.9</td>
<td>0.03262</td>
<td>586.2</td>
<td>0.10000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F5-T21</td>
<td>205188</td>
<td>480.7</td>
<td>0.002352</td>
<td>491.0</td>
<td>0.002503</td>
<td>501.4</td>
<td>0.011188</td>
<td>543.4</td>
<td>0.03293</td>
<td>585.4</td>
<td>0.10250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F6-L23</td>
<td>200000</td>
<td>491.0</td>
<td>0.004455</td>
<td>491.0</td>
<td>0.01140</td>
<td>533.9</td>
<td>0.03039</td>
<td>576.7</td>
<td>0.09750</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F6-L24</td>
<td>201706</td>
<td>474.5</td>
<td>0.002352</td>
<td>487.0</td>
<td>0.002457</td>
<td>499.5</td>
<td>0.01045</td>
<td>542.9</td>
<td>0.03052</td>
<td>586.3</td>
<td>0.09500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>201724</td>
<td>477.6</td>
<td>0.002352</td>
<td>489.0</td>
<td>0.002480</td>
<td>498.9</td>
<td>0.01161</td>
<td>541.3</td>
<td>0.03162</td>
<td>583.7</td>
<td>0.09875</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal</td>
<td>200000</td>
<td>464.3</td>
<td>0.002322</td>
<td>474.7</td>
<td>0.002453</td>
<td>485.0</td>
<td>0.01161</td>
<td>526.2</td>
<td>0.03162</td>
<td>567.4</td>
<td>0.09875</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Data from strain gage.
** Data from extensometer.

Table 3.6 Stress-strain model data for 6 mm coupons

<table>
<thead>
<tr>
<th>Coupon</th>
<th>E (MPa)</th>
<th>A* (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>B* (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>C* (or D) (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
<th>E** (MPa)</th>
<th>( \varepsilon ) (mm/mm)</th>
</tr>
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<tbody>
<tr>
<td>W3-T12</td>
<td>200000</td>
<td>211.0</td>
<td>0.001055</td>
<td>334.9</td>
<td>0.001966</td>
<td>458.9</td>
<td>0.004295</td>
<td>567.0</td>
<td>0.01887</td>
</tr>
<tr>
<td>W4-L15</td>
<td>200000</td>
<td>209.9</td>
<td>0.001050</td>
<td>336.4</td>
<td>0.001966</td>
<td>462.9</td>
<td>0.004314</td>
<td>581.3</td>
<td>0.01960</td>
</tr>
<tr>
<td>W4-T18</td>
<td>200000</td>
<td>229.4</td>
<td>0.001147</td>
<td>346.8</td>
<td>0.001991</td>
<td>464.1</td>
<td>0.004321</td>
<td>573.5</td>
<td>0.02045</td>
</tr>
<tr>
<td>W3-L26</td>
<td>200000</td>
<td>209.8</td>
<td>0.001049</td>
<td>338.9</td>
<td>0.001954</td>
<td>468.0</td>
<td>0.004340</td>
<td>575.6</td>
<td>0.01986</td>
</tr>
<tr>
<td>W3-T27</td>
<td>200000</td>
<td>213.3</td>
<td>0.001066</td>
<td>338.6</td>
<td>0.001966</td>
<td>463.8</td>
<td>0.004319</td>
<td>573.1</td>
<td>0.01973</td>
</tr>
<tr>
<td>Average</td>
<td>200000</td>
<td>214.7</td>
<td>0.001073</td>
<td>339.2</td>
<td>0.001969</td>
<td>463.5</td>
<td>0.004317</td>
<td>574.3</td>
<td>0.01970</td>
</tr>
<tr>
<td>Nominal</td>
<td>200000</td>
<td>224.6</td>
<td>0.001123</td>
<td>354.8</td>
<td>0.002018</td>
<td>485.0</td>
<td>0.004425</td>
<td>600.7</td>
<td>0.01970</td>
</tr>
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</table>

* Data from strain gage.
** Data from extensometer.
### Table 3.7 Material properties used in FE models

<table>
<thead>
<tr>
<th>Flange</th>
<th>Web</th>
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<tr>
<td><strong>Point</strong></td>
<td><strong>σ_p (MPa)</strong></td>
</tr>
<tr>
<td>A</td>
<td>465.4</td>
</tr>
<tr>
<td></td>
<td>469.6</td>
</tr>
<tr>
<td></td>
<td>473.7</td>
</tr>
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<td>477.9</td>
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<td>482.1</td>
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<td></td>
<td>485.2</td>
</tr>
<tr>
<td></td>
<td>486.2</td>
</tr>
<tr>
<td>C</td>
<td>487.1</td>
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<tr>
<td>D</td>
<td>490.6</td>
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<tr>
<td></td>
<td>612.4</td>
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<td>623.5</td>
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### Table 3.8 Welding process efficiency factor (after Young 1974)

<table>
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<th>Process</th>
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<tbody>
<tr>
<td>Submerged arc</td>
<td>0.90</td>
</tr>
<tr>
<td>Cored wire CO₂</td>
<td>0.85</td>
</tr>
<tr>
<td>Manual</td>
<td>0.80</td>
</tr>
<tr>
<td>Fusearc</td>
<td>0.75</td>
</tr>
<tr>
<td>MIG (spray)</td>
<td>0.62</td>
</tr>
<tr>
<td>MIG (dip)</td>
<td>0.42</td>
</tr>
<tr>
<td>Electro-slag</td>
<td>0.20</td>
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### Table 3.9 Flange FE meshes used with ECCS residual stresses

<table>
<thead>
<tr>
<th>Case</th>
<th>$l_{fc,E}$</th>
<th>$l_{w,E}$</th>
<th>N1</th>
<th>$l_{w,FE}$</th>
<th>N2</th>
<th>$l_{tc}$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mm)</td>
<td>(mm)</td>
<td></td>
<td>(mm)</td>
<td></td>
<td>(mm)</td>
<td></td>
</tr>
<tr>
<td>NC9</td>
<td>15.4</td>
<td>17.8</td>
<td>6</td>
<td>20.8</td>
<td>8</td>
<td>23.2</td>
<td>28</td>
</tr>
<tr>
<td>NC14</td>
<td>16.2</td>
<td>16.1</td>
<td>6</td>
<td>20.8</td>
<td>8</td>
<td>23.0</td>
<td>28</td>
</tr>
<tr>
<td>NC53</td>
<td>14.5</td>
<td>19.2</td>
<td>8</td>
<td>22.5</td>
<td>5</td>
<td>21.7</td>
<td>24</td>
</tr>
<tr>
<td>NC70</td>
<td>15.4</td>
<td>17.8</td>
<td>8</td>
<td>28.1</td>
<td>5</td>
<td>15.7</td>
<td>24</td>
</tr>
<tr>
<td>NC116</td>
<td>17.0</td>
<td>14.8</td>
<td>8</td>
<td>23.4</td>
<td>5</td>
<td>29.8</td>
<td>24</td>
</tr>
<tr>
<td>NC132</td>
<td>17.0</td>
<td>14.8</td>
<td>8</td>
<td>28.1</td>
<td>5</td>
<td>25.1</td>
<td>24</td>
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<td>16.2</td>
<td>15.7</td>
<td>10</td>
<td>27.0</td>
<td>3</td>
<td>18.5</td>
<td>22</td>
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<tr>
<td>NC93</td>
<td>16.2</td>
<td>15.7</td>
<td>10</td>
<td>27.0</td>
<td>3</td>
<td>35.2</td>
<td>22</td>
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</table>

### Table 3.10 Initial tensile stresses

<table>
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<tr>
<th>Case</th>
<th>$\sigma_{\text{ts}/0}$</th>
<th>$\sigma_{\text{tw}/0}$</th>
<th>$\sigma_{110}$</th>
<th>$\sigma_{220}$</th>
<th>$\sigma_{120}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
</tr>
<tr>
<td>NC9</td>
<td>511.9</td>
<td>449.9</td>
<td>389.7</td>
<td>129.9</td>
<td>225.0</td>
</tr>
<tr>
<td>NC14</td>
<td>513.7</td>
<td>410.3</td>
<td>355.5</td>
<td>118.5</td>
<td>205.3</td>
</tr>
<tr>
<td>NC53</td>
<td>516.0</td>
<td>459.8</td>
<td>367.7</td>
<td>207.3</td>
<td>276.1</td>
</tr>
<tr>
<td>NC70</td>
<td>520.2</td>
<td>359.5</td>
<td>311.4</td>
<td>103.8</td>
<td>179.8</td>
</tr>
<tr>
<td>NC116</td>
<td>516.0</td>
<td>342.8</td>
<td>296.9</td>
<td>99.0</td>
<td>171.4</td>
</tr>
<tr>
<td>NC132</td>
<td>517.0</td>
<td>295.9</td>
<td>236.7</td>
<td>133.4</td>
<td>177.7</td>
</tr>
<tr>
<td>NC88</td>
<td>522.3</td>
<td>331.9</td>
<td>265.4</td>
<td>149.6</td>
<td>199.3</td>
</tr>
<tr>
<td>NC93</td>
<td>515.1</td>
<td>322.5</td>
<td>257.9</td>
<td>145.4</td>
<td>193.6</td>
</tr>
</tbody>
</table>
Figure 3.1 Deformed shape of weak axis bending

Figure 3.2 Effects of $h_r/b_f$ on moment of inertia $I_z$
Figure 3.3 End cross section with bearing stiffener

Figure 3.4 Elastic buckling and nonlinear analysis results for NC70
Figure 3.5 Elastic buckled shape of NC70 with two corrugations
Figure 3.6 Elastic buckled shape of NC70 with four corrugations
Figure 3.7 Elastic buckled shape of NC70 with six corrugations
Figure 3.8 First three elastic buckled shapes of NC70C14
Figure 3.9 Top flange lateral displacement of three initial imperfection cases for NC70C14

Figure 3.10 Moment-rotation from nonlinear load deflection analyses for NC70C14
Figure 3.11 Initial geometric imperfection shapes investigated for NC70C6

(a) Imperfection shape EM1 based on elastic buckling mode 1

(b) Imperfection shape SD1

(c) Imperfection shape SD2
Figure 3.12 Nonlinear analyses using alternate initial imperfections
Figure 3.13 Unsupported widths of top flange
Figure 3.14 Typical post-peak deformations for short or medium span CWG
Figure 3.15 Moment versus rotation for NC93C2
(a) Peak moment (inc6) isometric view

(b) Peak moment (inc6) top view

Figure 3.16 Deformed shapes for NC93C2

156
(c) 90% peak moment post-peak (inc19) isometric view

(d) 90% peak moment post-peak (inc19) top view

Figure 3.16 Deformed shapes for NC93C2 (continued)
Figure 3.17 Convention of directions

Figure 3.18 Locations studied for flange local buckling
(a) Axial strain on south side

(b) Axial strain on north side

Figure 3.19 Flange plate bending at cross section L1
Figure 3.19 Flange plate bending at cross section L1 (continued)
Figure 3.20 Flange lateral bending strain and curvature at cross section L1

(a) Axial strain at middle surface

(b) Flange lateral bending curvature (part)
Figure 3.21 Area under high risk of flange local buckling

Integration points

Notation (US: upper surface, MS: mid-surface, LS: lower surface)

Figure 3.22 Yielded area at cross section L1
Figure 3.23 Stress states at the south side of cross section L3 of top flange
(c) Lower surface

Figure 3.23 Stress states at the south side of cross section L3 of top flange (continued)

Figure 3.24 Typical stress-strain curves for HPS 485W steel
Figure 3.25 Stress-strain curves from coupon tests

(a) 50 mm coupon

(b) 6 mm coupon
Figure 3.26 Stress-strain model (Salem 2004)

Figure 3.27 Stress-strain curves from coupon tests, stress-strain model and true stress-true strain based on stress-strain model (F6-L24)
Figure 3.28 Average stress-strain model with coupon test data for 50 mm plate
Figure 3.29 Shift average stress-strain curve to nominal yield stress
Figure 3.30 Average and nominal stress-strain models

(a) 50 mm plate

(b) 6 mm plate
Figure 3.31 Typical residual stresses in center-welded steel plate: (a) hot-rolled plate; (b) hot-rolled plate with flame-cut edges. (from Galambos 1998)

Figure 3.32 LTB strength of simply supported I-beams (from Trahair and Bradford 1998)
Figure 3.33 Center-welded flame-cut plate illustration (from Bjorhovde et al. 1971)
Figure 3.34 Measured residual stresses for the Fritz Plate

(1 ksi = 6.895 MPa, 1 inch = 25.4 mm)

Figure 3.35 Balanced average (BAS) and balanced equivalent (BES) residual stresses

(1 ksi = 6.895 MPa, 1 inch = 25.4 mm)
Figure 3.36 Finite element mesh for applying residual stresses to flanges of conventional I-girders

Figure 3.37 Initial stresses and resulting residual stresses
(a) Combination of 3- and 4-node elements

(b) Flange FE mesh adjacent to the inclined fold

Figure 3.38 Finite element mesh for applying residual stresses to CWG
Figure 3.38 Finite element mesh for applying residual stresses to CWG (continued)
Figure 3.39 Deformed shape under initial tensile stresses

Figure 3.40 Resulting axial normal residual stresses S11 for CWG on bottom flange middle surface
Figure 3.41 Initial tensile stresses and the resulting residual stresses for CWG

(a) Initial tensile stresses and the resulting residual stresses at section BF1

(b) Resulting residual stresses at section BF1 and BF2
Residual Stresses

Figure 3.42 Cases considered in the parametric study of NC88
Figure 3.43 Effects of initial geometric imperfection amplitude for NC88

Figure 3.44 Effects of steel stress-strain model for NC88
Figure 3.45 Comparison of Realistic and EPP stress-strain models

Figure 3.46 Effects of residual stress amplitude for NC88
(a) Residual stresses due to flame-cutting

(b) Residual stresses due to welding

(c) Total residual stresses

Figure 3.47 ECCS (1976) residual stresses for a flange plate (adapted from Barth 1996)
Figure 3.48 Effects of changing the half width of the weld tensile residual stress area

(a) Residual stresses on a typical cross-section

(b) Nonlinear load deflection analysis results
Figure 3.49 Effects of changing the width of the flame-cutting tensile residual stress area

(a) Residual stresses on a typical cross-section

(b) Nonlinear load deflection FE analyses results
Figure 3.50 Comparison of residual stress patterns ECCS and RS4 for NC88

Figure 3.51 Typical flange finite element meshes

(a) Notation
Figure 3.51 Typical flange finite element meshes (continued)
Figure 3.52 Moment versus left end rotation for NC88

Figure 3.53 Moment versus normalized top flange lateral displacement for NC88
Figure 3.54 FE analysis results and LTB strength formulas for NC9
Figure 3.55 FE analysis results and LTB strength formulas for NC14
Figure 3.56 FE analysis results and LTB strength formulas for NC53
Figure 3.57 FE analysis results and LTB strength formulas for NC70

(a) All data

(b) Inelastic region
Figure 3.58 FE analysis results and LTB strength formulas for NC88
Figure 3.59 FE analysis results and LTB strength formulas for NC93
Figure 3.60 FE analysis results and LTB strength formulas for NC116
Figure 3.61 FE analysis results and LTB strength formulas for NC132
Figure 3.62 The proposed formula with different torsional constants
Figure 3.62 The proposed formula with different torsional constants (continued)
Figure 3.62 The proposed formula with different torsional constants (continued)
Figure 3.62 The proposed formula with different torsional constants (continued)
Figure 3.63 Comparison of proposed nominal LTB capacity with AASHTO LRFD nominal LTB capacity

(a) NC9

(b) NC14
Figure 3.63 Comparison of proposed nominal LTB capacity with AASHTO LRFD nominal LTB capacity (continued)
Figure 3.63 Comparison of proposed nominal LTB capacity with AASHTO LRFD nominal LTB capacity (continued)
Figure 3.63 Comparison of proposed nominal LTB capacity with AASHTO LRFD nominal LTB capacity (continued)
4 Flange Lateral Bending under Vertical Load

When a corrugated web girder (CWG) is subjected to primary shear due to vertical load, the flanges of the CWG bend laterally due to torsion caused by the web eccentricity. This phenomenon was investigated by Abbas (2003) and detailed analytical solutions for the flange lateral bending behavior were developed for a CWG with sinusoidal corrugations. For CWGs with trapezoidal corrugations, a fictitious load approach was developed by Abbas (2003) to determine the flange lateral bending moment and bending stress. In this chapter, the results from the fictitious load approach are compared with finite element results for a CWG with typical bridge girder dimensions. The effects of intermediated lateral braces on the flange transverse bending of a simply supported CWG are also described. Finally, finite element analyses show that even under uniform primary bending, where shear is absent, flange lateral bending can also develop, and the flange lateral bending behavior under uniform primary bending is investigated.

4.1 Primary Shear Induced Flange Lateral Bending

4.1.1 Fictitious Load Approach Results

The fictitious load approach (Abbas 2003) is used to investigate the flange lateral bending behavior of a CWG with practical bridge girder dimensions. The fictitious load is a load that when applied laterally to the flange produces flange lateral bending similar to that of the flange of a CWG subjected to primary shear, from vertical load.

The CWG studied is case NC132 with 30 corrugations which gives a span to girder depth ratio of 27.9. The load case considered is a uniformly distributed load $p$ acting in the girder middle plane with a magnitude of 1 kN/mm. The vertical shear and moment distribution along the span are

$$V = \frac{pL}{2} \left(1 - \frac{2x}{L}\right)$$

$$M = \frac{px}{2} (L - x)$$

(4.1)

where $x$ is the distance of a cross-section from the left support and $L$ is the span length. The maximum shear and moment are $V_{\text{max}} = pL/2$ and $M_{\text{max}} = pL^2/8$. The fictitious load $p_f$ is determined as follows (Abbas 2003)

$$p_f = \frac{2}{h} V \tan \eta - \frac{2}{h} p e$$

(4.2)

where $h$ is the distance between the centroids of the top and bottom flange, $\eta$ is the slope of corrugation geometry with respect to the girder middle plane and is equal to the corrugation angle, $\alpha$, for the inclined fold and zero for the longitudinal fold, and $e$ is the eccentricity of the web from the girder middle plane. Using the results for $V$, $p_f$ can be written as
\[ p_t = \frac{2p}{h} \left( \left( \frac{L}{2} - x \right) \tan \eta - e \right) \]

By dividing both sides by \( 2p/h \), \( p_t \) can be normalized as follows
\[ p_m = \frac{p_h}{2p} = \left( \frac{L}{2} - x \right) \tan \eta - e \]

\( p_m \) can be divided into two components
\[ p_{mA} = \left( \frac{L}{2} - x \right) \tan \eta \]
\[ p_{mB} = e \]

The normalized fictitious load \( p_t \) and its components \( p_{mA} \) and \( p_{mB} \) for girder NC132C30 are shown in Figure 4.1. It can be seen that \( p_t \) is almost equal to \( p_{mA} \) and the contribution of \( p_{mB} \) is minor. The maximum value of \( p_{mA} \) is \( p_{mA\text{ max}} = L \tan \alpha/2 \) and the maximum value of \( p_{mB} \) is \( p_{mB\text{ max}} = h_r/2 \) and their ratio is
\[ \frac{p_{mA\text{ max}}}{p_{mB\text{ max}}} = \frac{L \tan \alpha}{h_r} = \frac{nL_c \tan \alpha}{h_r} \]

where \( n \) is the number of corrugations and \( L_c \) is the corrugaion length. For case NC132, the above ratio is approximately equal to 5\( n \) so that when the girder has a large number of corrugations, \( p_{mA} \) is dominant.

To determine the flange lateral bending moment and bending stresses due to primary shear, a single flange is modeled as a beam in its own plane and subjected to the fictitious load. Abbas (2003) did not study cases with intermediate lateral braces within the span. To study the effects of intermediate lateral braces, which are present in actual bridge girders, a number of intermediate supports are added to the single flange model used in the fictitious load analysis. Figure 4.2 shows several arrangements of lateral braces that were considered. For the case with 5 intermediate braces, the lateral unbraced length is 7.5 m which is a traditional cross-frame spacing in conventional steel I-girder bridges. Elastic analyses of these models under the fictitious load \( p_t \) were conducted and the flange lateral bending moment and lateral displacement are shown in Figure 4.3. The lateral bending moment and displacement have been normalized as
\[ M_m = \frac{M_h}{pL^2e_0} \]
\[ u_m = \frac{EIu_h}{pL^4e_0} \]

where \( M_m \) is the flange lateral bending moment, \( u_m \) is the flange lateral bending displacement, and \( e_0 = h_r/2 \). When no intermediate braces are included in the model, the lateral displacement is zero at the mid span. Therefore, the case with one intermediate brace was not analyzed. Figure 4.3(a) shows that without an intermediate
brace, the lateral bending moment oscillates along the span and each local peak corresponds to one corrugation. Both the moments and displacements are anti-symmetric about mid span which is expected when the CWG has an even number of half corrugations and is under uniformly distributed load (Abbas 2003). Due to this anti-symmetry, the flange lateral displacement at mid span is always zero so that the intermediate brace at this location has no effect.

The effects of the intermediate braces are illustrated in Figure 4.4 by comparing the lateral bending moments and displacements of two cases, namely, the case with no intermediate braces, and the cases with two intermediate braces. It can be seen that the bending moment (Figure 4.4(a)) is modified by the brace reactions, which are shown on the lateral displacement plot (Figure 4.4(b)). The brace reactions change the moment envelope but do not change the magnitude of the local moment variation. The intermediate braces greatly reduce the magnitude of the flange lateral displacement. Figure 4.3 shows that the maximum flange lateral displacement decreases with an increase in the number of intermediate braces. However, the intermediate braces do not necessarily reduce the flange lateral bending moment and the brace reactions appear to alter the moment envelope without substantially changing the maximum moment.

### 4.1.2 Comparison of Fictitious Load Approach and Finite Element Analysis

The results from the fictitious load approach are compared to the results of a finite element (FE) analysis of a CWG model. The case studied is NC132 with 30 corrugations (NC132C30). Initially, intermediate braces were not included in the analysis models. A uniformly distributed load was applied to the top and bottom flange of the FE model as a downward pressure load. The top flange normalized lateral displacements from the two approaches are shown in Figure 4.5. It can be seen that the results from the FE analysis are much smaller than those from the fictitious load approach. Since the FE model includes two end stiffeners, the possibility that the stiffeners had some effect on the lateral displacement was considered. The thickness of each end stiffeners was reduced by half, and it was found that the effect of the end stiffeners on the lateral displacement is minor.

As noted earlier, the flange lateral bending observed for CWGs under primary shear is due to torsion (Abbas 2003). The fictitious load approach assumes this torsion is entirely resisted by warping torsion and the resistance of St. Venant torsion is neglected. According to torsion theory for conventional I-girders, the St. Venant torsion resistance is more important for a long girder or a girder with a small $E I_w / K_T$ ratio where $E I_w$ and $K_T$ are the warping torsion and St. Venant torsion stiffness respectively. For such girders, the flange lateral moment and displacement from the fictitious load approach is larger than in reality because the St. Venant torsion resistance is neglected. To verify this finding, a shorter girder with six corrugations (NC132C6) was analyzed using both the fictitious load approach and FE analysis. The fictitious load for this case is shown in Figure 4.6. The top flange lateral displacements
are shown in Figure 4.7. It can be seen that for this shorter model, the results from the two analyses are close, confirming the above finding.

Cases for NC132C30 with intermediate braces were also studied and top flange lateral displacements for cases with two, three, four and five intermediate braces are shown in Figure 4.8. It can be seen that the differences between the fictitious load approach results and FE analysis results are reduced when there are more intermediate braces. For the case with five intermediate braces, which is a practical lateral brace arrangement for a bridge girder, the flange lateral displacements from the two analyses are close.

From above comparisons, it can be noted that for short girders and for girders with practical lateral unbraced lengths, the fictitious load approach provides useful results. To get more accurate results, the St. Venant torsion resistance should be considered, as follows.

By assuming vertical shear is entirely carried by the web, Abbas (2003) derived a relation for the torque \( M_x \) and the primary shear \( V_z \).

\[
\frac{dM_x}{d(V_z e)} = -2
\]

where \( e \) is the web eccentricity. Integrating the above equation once results in

\[
M_x = -2V_z e + C
\]

where \( V_z \) and \( e \) are functions of \( x \). This torque is resisted by both warping and St. Venant torsion resistance.

\[
M_x = M_w + M_{SV}
\]

So the previous equation can be rewritten as

\[
-2V_z e + C = -EI_w \phi^{(4)} + K_T \phi''
\]

where \( \phi \) is the angle of twist. Note for a CWG, the uniform torsion stiffness developed in Chapter 2 should be used for \( K_T \). Taking the derivative once with respect to \( x \) results in

\[
-2 \frac{d(V_z e)}{dx} = -EI_w \phi^{(4)} + K_T \phi''
\]

(4.3)

And this differential equation describes CWG torsion due to primary shear.

Equation (4.3) can be written also in terms of flange lateral bending displacement \( u_f \). Assuming the top and bottom flange are identical,

\[
\phi = \frac{2u_f}{h}
\]

As explained in Section 3.1, an approximation for the warping torsion constant is

\[
I_w = \frac{I_{sf} h^2}{2}
\]

where \( I_{sf} \) is the moment of inertia of the flange about its strong axis. This approximation is accurate for a thin-walled cross section and neglects any effect of the corrugations on \( I_w \). Substituting for \( \phi \) and \( I_w \) into Equation (4.3) results in
which is the differential equation that describes the flange lateral bending due to primary shear acting on a CWG. If the St. Venant torsion resistance is neglected, the second term on the right-hand side is neglected, and the fictitious load $p_t$ can be defined as

$$p_t = \frac{2 d(V_se)}{hx}$$

This is equivalent to Equation (4.2). Equation (4.4) then becomes

$$p_t = EJ_zu_t^{IV}$$

which is the basis of the fictitious load approach. A solution for either Equation (4.3) or (4.4) would then provide the flange lateral bending displacement, without neglecting St. Venant torsion resistance.

**4.1.3 Design Equation for Shear Induced Flange Lateral Bending Moment**

Figure 4.3 shows that the flange lateral bending moment varies along the CWG. For design purposes, the maximum flange lateral bending moment is of more interest than the actual variation. A design formula for the flange lateral bending moment for trapezoidal corrugations was provided by Sause et al. (2003).

$$M_{fl_{des}} = \frac{V_{ref}}{D} \left( b + \frac{d}{2} \right) h_f$$

(4.5)

where $b$, $d$, $h_f$, and $D$ are corrugated web geometric parameters defined in Figure 2.2 and $V_{ref}$ is a reference shear. Equation (4.5) is based on the primary shear induced flange lateral bending analyses conducted by Abbas (2003). For a simply supported CWG under uniformly distributed load with no intermediate braces, $\left( M_{fl_{des}} \right)$ agrees exactly with fictitious load approach results when $V_{ref}$ is set equal to $V$ from Equation (4.1), as shown in Figure 4.9. Sause et al. (2003) recommend that $V_{ref}$ should be the value of the factored design shear force envelope. To be conservative in the regions of maximum primary bending moment where the primary shear is small, $V_{ref}$ should not be less than 25% of maximum shear in the span (Sause et al. 2003). Using these recommendations, $\left( M_{fl_{des}} \right)$ was calculated and is compared in Figure 4.10 with the fictitious load approach for a simply supported CWG (case NC132C30) under uniformly distributed load. It can be seen that when intermediate braces are considered, $\left( M_{fl_{des}} \right)$ from Equation (4.5) is conservative. Note that the St. Venant torsion resistance is neglected in the derivation of Equation (4.5) as well as in the fictitious load approach results shown in Figure 4.10. Therefore Equation (4.5) should provide conservative estimates of the shear induced flange lateral bending moment.
4.2 Primary Moment Induced Flange Lateral Bending

A FE model of case NC93 with 100 corrugation was developed as part of the studies reported in Chapter 5. The CWG is simply supported with four intermediate braces so that the lateral unbraced length is $L/5$. A uniformly distributed load is applied and the CWG is assumed to be elastic. The top flange lateral bending moment from the FE analysis for half of the span is compared to $(M_{ni})_{des}$ in Figure 4.11. The figure shows that $(M_{ni})_{des}$ underestimates the flange lateral bending moment between the mid span and the 80th corrugation. Since this region is subject to a large primary moment and low primary shear, the possibility that the flange lateral bending moment is induced by primary bending moment was considered, and flange lateral bending induced by uniform primary bending moment (with no primary shear) was studied.

4.2.1 Flange Lateral Bending due to Uniform Primary Bending Moment

Two models from case NC88 under uniform primary bending are studied. One model has 9.5 corrugations and is designated as NC88C9.5. This model has an odd number of half corrugations. The other model has 10 corrugations and is designated as NC88C10. This model has an even number of half corrugations. The two models are subjected to uniform bending moment of magnitude $M_p$, which is the CWG plastic moment calculated by neglecting the contribution of web (Section 1.7), however, the behavior of each model is assumed to be elastic. No intermediate braces are included. The top flange lateral displacements of the two models are shown in Figure 4.12, where flange lateral bending under uniform bending moment can be observed. For an odd number of half corrugations (NC88C9.5), a small overall single curvature lateral displacement is observed along with more frequent variations with much larger magnitude. For an even number of half corrugations (NC88C10), a very small overall double curvature lateral displacement is observed along with more frequent variations with much larger magnitude.

Flange lateral bending under uniform bending moment is attributed to the deformation incompatibility of the flange and corrugated web under axial extension and compression. Figure 4.13 shows the deformation of one corrugation length of the flange and web under the same axial extension when they are not connected along the web-to-flange interface. It can be seen that the extension of the corrugated web mainly comes from the web plate bending deformation (Abbas 2003), which reinforces the assumption that the axial normal stress carried by a corrugated web can be neglected, as stated in Section 1.7. The dashed line in Figure 4.13(b) indicates the deformed position of the web if it were connected to the flange. To overcome this deformation incompatibility, the flange will have to push the corrugated web into the position shown by the dashed line. The reactions from the corrugated web will push on the flange in the directions shown by arrows in Figure 4.13(b), resulting in flange lateral bending. Figure 4.14 shows the deformation of one corrugation length of the flange and web under the same axial compression when they are not connected along their interface. A similar deformation incompatibility is observed, which, if overcome, will
push the flange in directions opposite to those under extension, as shown in Figure 4.14(b).

Figure 4.15 shows the deformation of a CWG under uniform bending. For clarity only the web and the bottom flange under tension are shown. The figure shows that at locations close to the flange, the web deformation is compatible with the flange deformation. Also shown on the plot are the directions in which the flanges are displaced laterally. The top and bottom flanges deform in opposite directions and the directions are consistent with those shown in Figure 4.13(b) and Figure 4.14(b).

Re-examining Figure 4.12(a), it can be seen that the top flange of NC88C9.5 is pushed downward over 10 half corrugations and upward over 9 half corrugations. There is a net force downward so there is also an overall single curvature deflection downward. Figure 4.12(b) shows that the top flange of NC88C10 is pushed both downward over 10 half corrugations and upward over 10 half corrugations. The distribution of these displacements is anti-symmetric about the mid span, so there is also an overall double curvature.

The flange lateral bending moment due to primary uniform bending moment depends primarily on the interactions between the web and flanges locally, and the overall bending is small and can be neglected. To show that the flange lateral bending moment does not depend on the span length, two models based on case NC93 are analyzed. One model is 10 corrugations long and the other model is 20 corrugations long. The top flange lateral bending moment of the two models from FE analysis is shown in Figure 4.16. Only the results for the first five corrugations are plotted. It can be seen that the flange lateral bending moments are the same for the two models, indicating that the flange lateral bending moment due to primary uniform bending moment is independent of span length. This flange lateral bending moment is proportional to the magnitude of the primary bending moment and is a function of the girder and the corrugation geometry.

### 4.2.2 Simplified Model for Flange Lateral Bending due to Uniform Primary Moment

From above analysis results, it can be concluded that Equation (4.5) will be unconservative for cases with low primary shear but high primary moment since it does not consider the flange lateral bending due to primary bending moment. As discussed earlier, the maximum moment is of interest for design. It can be seen from Figure 4.16 that the maximum flange lateral bending moment occurs at the middle of the longitudinal fold. A simplified expression to calculate the maximum bending moment at this location is developed as follows. First, it is assumed that the primary bending moment is carried by flanges only (see Section 1.7), as shown in Figure 4.17(a), so that the normal force acting on the flange centroid due to the primary moment is

\[ P = \frac{M}{h} \]

where \( M \) is the primary moment and \( h \) is the distance between the flange centroids. Next, a small portion of the web is assumed to act with the flange as an assumed cross section, under the force \( P \), which acts at the flange centroid, as shown in Figure...
4.17(b). Due to the presence of this portion of the web, the centroid of the assumed cross section is eccentric from the centroid of the flange. As a result, force P produces a bending moment about the centroid of the assumed cross section. The position of centroid of the assumed cross-section with respect to the left edge of the flange is determined as follows

\[ y_0 = \frac{A_b b_f + A_w (b_f - h_w)}{2 (A_f + A_{wx})} \] (4.6)

where \( A_f = b_f t_f \) and \( A_{wx} = xt_w \) with \( x \) as defined in Figure 4.17(b). The flange lateral bending moment is considered to be due to this eccentricity

\[ M_t = P \left( \frac{b_f}{2} - y_0 \right) = \frac{M}{h} \left( \frac{b_f}{2} - y_0 \right) \] (4.7)

which is a function of \( x \). The value of \( x \) can be determined by setting \( M_t \) equal to the maximum \( M_t \) from the FE analyses under uniform primary bending moment, \( M_{\text{FE}} \), as follows

\[ \frac{M}{h} \left( \frac{b_f}{2} - y_0 \right) = M_{\text{FE}} \]

and solving for \( x \). For example, for NC93C20, the FE results are shown in Figure 4.16. The applied uniform primary bending moment equals the plastic moment of the cross section composed of the flanges alone \( M_p = 3.71 \times 10^7 \) kN-mm. The maximum flange lateral bending moment from the FE analysis is \( M_{\text{FE}} = 4.669 \times 10^4 \) kN-mm. The other parameters are \( b_f = 500 \) mm, \( t_f = 50 \) mm, \( t_w = 9 \) mm, \( h = 2550 \) mm and \( h_w = 270.2 \) mm. The location of the centroid of the assumed section is

\[ y_0 = \frac{1.8 \times 10^7 + 2968.2x}{60000 + 18x} \]

and solving this equation results in \( x = 81.3 \) mm which is \( 3.2\% \) of \( h \), the distance between flange centroids.

Similarly, \( x \) can be determined for other cases. Table 4.1 shows the results for several selected cases. For simplicity, \( 5\% \) of \( h \) will be considered in the simplified flange lateral bending model. Further work on this topic is needed to develop a more rigorous approach for determining \( x \).

**4.2.3 Modified Design Equation for Flange Lateral Bending Moment**

Equation (4.5) can be modified to include the contribution of the primary bending moment induced flange lateral bending moment, as well as the primary shear induced flange lateral bending moment as follows
where the first term on the right-hand side is the contribution of the primary shear and the second term is the contribution of the primary bending moment. \( y_0 \) is defined by Equation (4.6), and assuming \( x = 0.05h \), \( y_0 \) can be rewritten as

\[
y_0 = \frac{b_f t_f + 0.05h w (b_f - h_r)}{2(b_f t_f + 0.05h w)}
\]

Note in Equation (4.8) \( V_{ref} \) should be the value of the factored primary shear force envelope. The requirement that \( V_{ref} \) should not be less than 25% of the maximum shear in the span can be eliminated. \( M \) should be the value of the factored primary bending moment envelope. The flange lateral bending moment from finite element analysis of model NC93C100 with four intermediate braces (Figure 4.11) is compared with results from Equation (4.8) in Figure 4.18. It can be seen that Equation (4.8) is very conservative.

The conservatism has three sources. First, the percentage of web assumed to be acting with the flanges is conservatively estimated which results in a larger flange lateral moment due to primary bending. Second, the calculated effect of the primary shear did not consider the St. Venant torsion resistance as discussed earlier. Third, within one corrugation length, the location of the maximum flange lateral bending moment due to primary shear does not coincide with the location of the maximum flange lateral bending moment due to primary moment while the design equation always adds the maxima of the two together. Although Equation (4.8) produces a conservative estimate of the flange lateral bending moment, it is proposed for design purposes until a more accurate formula is available.

### 4.3 Summary

The fictitious load approach (Abbas 2003) was used to study the primary shear induced flange lateral bending moment in CWGs in the presence of intermediate braces. It was found that the brace forces change the overall pattern of the flange lateral bending moment over the CWG span while local variation of the flange lateral bending moment does not change. It was also found that the flange lateral displacement decreases as more intermediate braces are introduced. A comparison of the flange lateral displacements from the fictitious load approach and FE analysis of a CWG shows that the fictitious load approach overestimates the flange lateral displacement. This result is attributed to the fact that the fictitious load approach does not consider the St. Venant torsion resistance.

FE analysis results also show that the previously proposed design formula to estimate the flange lateral bending moment from primary shear underestimates the flange lateral bending moment in regions of low primary shear but high primary bending moment. FE analysis results showed that the primary bending moment induces flange lateral bending, which is the result of a deformation incompatibility of the flange and corrugated web under axial extension or axial compression. A simple
formula is proposed to calculate the flange lateral bending moment due to the primary bending moment.

A modified design formula is proposed which considers the effects of both primary shear and primary bending moment induced flange lateral bending moment. A comparison with FE analysis results shows that the modified design formula is conservative.
Table 4.1 Percentage of web to include for simplified flange lateral bending model

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<th>$h$ (mm)</th>
<th>$x/h$ (%)</th>
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<td>2.4</td>
</tr>
<tr>
<td>NC14</td>
<td>62.8</td>
<td>2550</td>
<td>2.5</td>
</tr>
<tr>
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<td>59.0</td>
<td>2040</td>
<td>2.9</td>
</tr>
<tr>
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<td>80.0</td>
<td>1545</td>
<td>5.2</td>
</tr>
<tr>
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<td>80.6</td>
<td>2050</td>
<td>3.9</td>
</tr>
<tr>
<td>NC93</td>
<td>82.8</td>
<td>2550</td>
<td>3.3</td>
</tr>
<tr>
<td>NC116</td>
<td>83.5</td>
<td>1555</td>
<td>5.4</td>
</tr>
<tr>
<td>NC132</td>
<td>81.8</td>
<td>1555</td>
<td>5.3</td>
</tr>
</tbody>
</table>
Figure 4.1 Fictitious load for NC132C30
Figure 4.1 Fictitious load for NC132C30 (continued)
Figure 4.2 NC132C30 lateral brace arrangements
Figure 4.3 Flange lateral bending moment and displacement under fictitious load for NC132C30

(a) No intermediate brace
(b) Two intermediate braces

Figure 4.3 Flange lateral bending moment and displacement under fictitious load for NC132C30 (continued)
(c) Three intermediate braces
Figure 4.3 Flange lateral bending moment and displacement under fictitious load for NC132C30 (continued)
(d) Four intermediate braces
Figure 4.3 Flange lateral bending moment and displacement under fictitious load for NC132C30 (continued)
(e) Five intermediate braces
Figure 4.3 Flange lateral bending moment and displacement under fictitious load for NC132C30 (continued)
Figure 4.4 Effects of intermediate braces

(a) Normalized flange lateral bending moment

(b) Normalized flange lateral displacement
Figure 4.5 Flange lateral displacement comparison for NC132C30

Figure 4.6 Fictitious load for NC132C6
Figure 4.7 Flange lateral displacement comparison for NC132C6
Figure 4.8 Comparison of top flange lateral displacement for NC132C30

(a) Two intermediate braces

(b) Three intermediate braces
Figure 4.8 Comparison of top flange lateral displacement for NC132C30 (continued)

(c) Four intermediate braces

(d) Five intermediate braces
Figure 4.9 NC132C30 with no intermediate brace

(a) No intermediate brace

Figure 4.10 Flange lateral bending moment from fictitious load approach and Equation 4.5
Figure 4.10 Flange lateral bending moment from fictitious load approach and Equation 4.5 (continued)
Figure 4.10 Flange lateral bending moment from fictitious load approach and Equation 4.5 (continued)
Figure 4.11 Flange lateral bending moment of second half of NC93C100
Figure 4.12 Top flange lateral displacements under uniform bending for NC88
Figure 4.13 Flange and corrugated web deformation under axial extension (NC88)
Figure 4.14 Flange and corrugated web deformation under axial compression (NC88)
Figure 4.15 Corrugated web local deformation under uniform bending (NC88)

Figure 4.16 Top flange lateral bending moment under uniform bending for NC93
Figure 4.17 Simplified flange lateral bending model due to uniform bending

Figure 4.18 Flange lateral bending moment and modified design equation for NC93C100 with 4 intermediate braces
5 Lateral Torsional Buckling Under Moment Gradient Bending

The lateral torsional buckling (LTB) strength of corrugated web girders (CWGs) under uniform bending moment was presented in Chapter 3. For a real bridge girder, however, the bending moment is not uniform over a lateral unbraced length due to the vertical loads within the girder span. The non-uniform moment condition is termed moment gradient bending. In this chapter, the LTB strength of CWGs under moment gradient bending is presented. First, nonlinear load deflection analyses of finite element (FE) models are used to study the interaction of the vertical load induced flange lateral displacement and the initial geometric imperfection. Second, FE models for LTB under moment gradient bending are developed based on simply supported CWGs with multiple intermediate lateral braces under uniformly distributed load. Third, the LTB strength of CWGs under moment gradient bending is studied using elastic buckling analyses and nonlinear load deflection analyses. Finally, the LTB strengths of CWGs and conventional I-girders under both moment gradient bending moment and uniform bending moment are compared.

5.1 Finite Element Modeling

5.1.1 Initial Imperfections

In the study of LTB under uniform bending, initial geometric imperfections were introduced in FE models to account for the effects of girder out-of-straightness due to fabrication. These initial geometric imperfections are a major factor in determining the LTB strength under uniform bending. For moment gradient bending, the compression flange lateral displacement induced by the vertical load could be significant, as discussed below, and, therefore, should be considered along with the initial geometric imperfections.

To investigate how the vertical load induced compression flange lateral displacement compares with the flange initial lateral displacement due to initial geometric imperfection, FE models of case NC88 with spans of 4, 8 and 12 corrugations were analyzed. The three FE models are subjected to a concentrated moment $M_{end}$ at the right end which has a magnitude equal to the plastic moment $M_p$, as illustrated in Figure 5.1. Elastic analyses were used to find the compression flange lateral displacement of the FE models. The results are shown in Figure 5.2, where the lateral displacement $u_t$ is normalized by the girder length $L$ and the position $x$ is also normalized by $L$. It can be noted that the normalized displacement decreases with an increase in span length and the displacement is comparable in magnitude to the flange lateral displacement of the initial geometric imperfections used previously for the uniform bending cases discussed in Chapter 3, which have a maximum magnitude of $L/1000$. It should be mentioned that the vertical load induced flange lateral displacement increases with the increasing load. When the maximum applied moment
is smaller than $M_p$, the corresponding flange lateral displacement will be smaller than that shown in Figure 5.2.

The initial imperfection of the compression flange could be in the same direction or in the opposite direction of the vertical load induced lateral displacement. When they are in the same direction, the effect is equivalent to that of a larger initial geometric imperfection, and the LTB strength of the girder may be reduced. When they are in opposite directions, the effect is equivalent to that of a reduced initial geometric imperfection and the LTB strength of the girder may be increased.

This issue is investigated by studying the LTB strength of case NC88 under the vertical load shown in Figure 5.1. First, the elastic LTB strength is determined by linear elastic buckling analysis. Then, the inelastic LTB strength is determined by nonlinear load deflection analysis. For the purpose of this investigation, residual stresses were not considered. The magnitude of the initial geometric imperfection is equal to either $L/1000$ or $-L/1000$. When the magnitude of the initial geometric imperfection is equal to $L/1000$, initial geometric imperfection and vertical load induced compression flange lateral displacement are in the same direction. When the magnitude of the initial geometric imperfection is equal to $-L/1000$, the initial geometric imperfection and vertical load induced compression flange lateral displacement are in the opposite direction.

The nonlinear load deflection analysis results show that the FE model of NC88 with an unbraced length of two corrugations failed in shear and the direction of the initial geometric imperfection did not affect the peak moment. The results for the models with longer unbraced lengths are shown in Figure 5.3 where the LTB strengths are normalized by $M_p$. It can be seen that when initial geometric imperfection and vertical load induced compression flange lateral displacement are in the same direction (Same), the LTB strength is lower than when they are in opposite directions (Opposite). The reduction in LTB strength is the largest for the FE model with eight corrugations (NC88C8) and is smaller for models with longer or shorter lateral unbraced lengths. For models with longer lateral unbraced lengths, the vertical load induced flange lateral displacements are smaller, as shown in Figure 5.2. For models with shorter lateral unbraced lengths, though the vertical load induced flange lateral displacement are larger, the LTB behavior is not strongly influenced by the effects of flange lateral displacement. It can be seen from Figure 5.3 that when the initial geometric imperfection and the vertical load induced compression flange lateral displacements are in opposite directions, the LTB strength is close to either the elastic LTB strength or $M_p$.

From above analysis, it can be concluded that care should be taken in establishing the direction of the initial geometric imperfections so that the nonlinear load deflection analyses do not overestimate the LTB strength.
5.1.2 Development of Study Cases for LTB under Moment Gradient Bending

For the study of LTB under moment gradient bending, the FE models are based on simply supported CWGs with multiple intermediate lateral braces under uniformly distributed load. From the simply supported CWGs with multiple unbraced lengths, two unbraced lengths are selected for the study of LTB under moment gradient bending. FE models of these isolated unbraced lengths are developed for use in the linear elastic buckling analyses and the nonlinear load deflection analyses.

For conventional steel I-girders, the LTB strength depends on the moment gradient. For CWGs, it is expected that the LTB strength also depends on the moment gradient. The moment gradient is quantified by the so-called moment gradient factor $C_b$ which is defined in AISC LRFD Specifications (1999) as

$$ C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C} \quad (5.1) $$

where $M_{\text{max}}$ is the maximum primary moment of the unbraced segment. $M_A$, $M_B$ and $M_C$ are the primary moment at $\frac{1}{4}$, midpoint and $\frac{3}{4}$ point of the unbraced segment. Absolute values are used for all the bending moments in Equation (5.1).

For all cases studied, the simply supported CWG has four equally spaced intermediate lateral braces within the span so that there are five lateral unbraced segments of equal length in the span. The five unbraced segments are identified as $L_{b1}$, $L_{b2}$, $L_{b3}$, $L_{b4}$ and $L_{b5}$. The moment gradient factors, $C_b$, for each unbraced segment are calculated and shown in Figure 5.4. It is important to recognize that $C_b$ does not change with the span length as long as the arrangement of the intermediate lateral braces and the loading condition (uniformly distributed load) do not change. From the lateral brace arrangement shown in Figure 5.4, the unbraced segment $L_{b1}$ has the highest moment gradient and was selected for a detailed study of the LTB strength of CWGs under a realistic moment gradient bending condition. The unbraced segment $L_{b3}$ has a moment gradient which is small and the loading of $L_{b3}$ is close to uniform bending. The results for $L_{b3}$ were used to verify the results for LTB strength under uniform bending from Chapter 3.

Unbraced segments $L_{b1}$ and $L_{b3}$ were modeled as isolated segments, with simple supports. The applied loads on these FE models are shown in Figure 5.5. The applied loads include the primary moment at the ends, $M_{\text{end}}$, the flange transverse moment at the ends, $M_{f\text{-end}}$, and the uniformly distributed load $p$. These applied loads are kept proportional to one another, with their relative magnitudes and their directions determined from linear elastic analysis of an FE model of the full CWG, including all five lateral unbraced lengths, under uniformly distributed load. The flange lateral bending moment $M_{f\text{-end}}$ at the ends of $L_{b1}$ and $L_{b3}$ were determined from the elastic analysis of the full CWG.
The distributed load \( p \) was applied to both flanges with each flange carrying half of the total distributed load. The flange may bend due to the uniformly distributed load, as illustrated in Figure 5.6, which may influence the nonlinear load deflection analysis results. This issue was investigated by considering two methods of applying the uniformly distributed load: (1) to the full width of the flange and (2) to the area of the flanges between the two longitudinal web folds but keep the total vertical force unchanged (see Figure 5.6). Linear elastic analyses showed that the flange vertical deflection is reduced when the uniformly distributed load is applied to the area of the flanges between the two longitudinal web folds. However, nonlinear load deflection analyses showed that the peak moment (LTB strength) is not affected by these flange vertical deflections. For convenience, then, the uniformly distributed load is applied to the full width of both flanges. These are the loading conditions used for both the linear buckling analyses and the nonlinear load deflection analyses.

To correctly apply the initial geometric imperfection, as discussed in Section 5.1.1, the compression flange lateral displacement in unbraced segments \( L_{b1} \) and \( L_{b3} \) were determined from an elastic analysis of the full CWG under a uniformly distributed load. For this analysis, a model of case NC88 with 40 corrugations was analyzed so that each of the five lateral unbraced segments each have eight corrugations. The compression flange lateral displacements in unbraced segments \( L_{b1} \) and \( L_{b3} \) are shown in Figure 5.7. It can be seen that unbraced segment \( L_{b1} \) has a large single curvature lateral displacement with relatively small secondary variations. Unbraced segment \( L_{b3} \) has a small double curvature lateral displacement with relatively large secondary variations. The flange lateral displacement of segment \( L_{b1} \) is much larger that that of \( L_{b3} \).

Elastic analyses of the FE models of the isolated lateral unbraced segments were conducted to determine the compression flange lateral displacements and compare them with those from the elastic analysis of the model of the full CWG. Compression flange lateral displacement from the FE models of the isolated lateral unbraced segments and the full CWG are compared in Figure 5.7. The flange lateral displacements shown in Figure 5.7 are scaled to a level corresponding to the primary bending moment at the ends of the unbraced lengths, \( M_{\text{end}} \), equal to \( M_p \). It can be seen that the compression flange lateral displacements from the two FE models compare very well.

The compression flange lateral displacements are the results of three loads: the uniformly distributed load \( p \), the primary bending moment applied at the ends of the unbraced segment, \( M_{\text{end}} \), and the flange lateral bending moment applied at the ends of the unbraced segment \( M_{\ell \text{-end}} \), as illustrated in Figure 5.5. To observe how each of these three loads affects the flange lateral displacement, the lateral displacements due to each of these three loads were determined from elastic analyses of the FE models of the isolated lateral unbraced segments. The results are shown in Figure 5.8, Figure 5.9 and Figure 5.10. Figure 5.8(a), Figure 5.9(a), and Figure 5.10(a) show that the compression flange lateral displacement in unbraced segment \( L_{b1} \) is mainly caused by
the primary bending moment at the end (and the related primary shear). Figure 5.8(b), Figure 5.9(b), and Figure 5.10(b) show that the lateral displacement in unbraced segment \( L_{b3} \) is mainly caused by the uniformly distributed load (and the related primary shear). The flange lateral bending moment \( M_{t,\text{end}} \) reduces the compression flange lateral displacement in both unbraced segments. Figure 5.9(b) shows that the secondary displacement variation in segment \( L_{b3} \) is caused by the primary bending moment.

Figure 5.11 compares the initial geometric imperfection with the total compression flange lateral displacements induced by vertical loads combined with the initial geometric imperfection. The flange lateral displacements due to vertical loads correspond to \( M_{\text{end}} \) equal to \( p \). The initial geometric imperfection of the compression flange over the unbraced length is a half sine function with a maximum magnitude of \( L_{b3}/1000 \). For unbraced segment \( L_{b1} \), the combined lateral displacement is significantly larger than the initial geometric imperfection and the combined lateral displacement can be expected to reduce the LTB strength relative to a LTB strength based on the initial geometric imperfection alone. For unbraced segment \( L_{b3} \), the compression flange lateral displacement due to vertical load is very small compared to the initial geometric imperfection.

### 5.2 Lateral Torsional Buckling Analysis Results

As discussed in Section 5.1.2, the LTB strengths of CWGs under moment gradient bending were determined by both elastic buckling analyses and nonlinear load deflection analyses of FE models of isolated unbraced segments. Three cases were selected to study the LTB strength of unbraced segment \( L_{b1} \), namely cases NC53, 88 and 93. Cases NC14 and 93 were selected for the study of unbraced segment \( L_{b3} \). The loading, defined in Figure 5.5, for each of these cases is summarized in Table 5.1 and Table 5.2, where \( p \) is the total uniformly distributed load, and \( M_{t,\text{end}} \) is the top flange lateral bending moment at the end, which is determined from elastic analyses of FE models of the corresponding full CWGs under uniformly distributed load. Both \( p \) and \( M_{t,\text{end}} \) are scaled to correspond to a primary bending moment, \( M_{\text{end}} \), equal to \( p \).

Elastic buckling analyses were conducted to determine the elastic LTB strength of these FE models. The inelastic LTB strength was determined from nonlinear load deflection analyses. The FE meshes of these models are the same as those developed for the study of CWGs under uniform bending (see Section 3.5.1). For the nonlinear load deflection analyses, the FE models use the realistic stress-strain model, an initial imperfection with an amplitude of \( L/1000 \), and the residual stress model based on ECCS formulas. Residual stresses are developed using the same approach used for the CWGs under uniform bending moment.
5.2.1 Finite Element Analysis Results for Segment $L_{b1}$

The load versus deflection curves from nonlinear load deflection analyses of cases NC53, 88 and 93 are shown in Figure 5.12, Figure 5.13 and Figure 5.14, respectively. The primary moment at the right end, $M_{end}$, versus right end rotation plots indicate the in-plane behavior while the primary moment versus flange lateral rotation plots indicate the out-of-plane behavior. It can be seen that the in-plane behavior is similar to what is observed for LTB under uniform bending, that is, the pre-peak softening of the moment versus end rotation plot is small for girders with certain lateral unbraced lengths, for example, C10, C12 and C14 for case NC53, but is large for girders with shorter or longer lateral unbraced lengths. Also generally speaking, longer unbraced lengths have larger out-of-plane deformation at the peak moment.

Figure 5.15 shows the peak primary bending moment versus the lateral unbraced length together with the elastic buckling analysis results and the peak primary moment under uniform bending. The moment gradient bending results are labeled as MG and the uniform bending results are labeled as UM. It can be seen that the peak moment under moment gradient bending is significantly larger than the peak moment under uniform bending.

5.2.2 Comparison with Proposed Design Formula for Segment $L_{b1}$

The results presented in Chapter 4 show that flange lateral bending occurs when a CWG is subjected to primary shear and/or primary bending moment from vertical load. To consider the effects of flange lateral bending, a combined moment demand is introduced, based on the approach of the AASHTO LRFD Bridge Design Specifications (2004). Then, the corresponding LTB strength for CWGs is defined.

AASHTO LRFD Bridge Design Specifications (2004) define a combined moment demand to address the effect of simultaneous primary bending and flange lateral bending. Similarly, to consider the effects of flange lateral bending in CWGs induced by vertical load, a combined moment demand $M_{u,\text{comb}}$ is defined for a CWG under vertical load as follows

$$M_{u,\text{comb}} = M_u + M_{ul}. \tag{5.2}$$

where $M_u$ is the actual primary moment from an analysis of the CWG under vertical load and $M_{ul}$ is the notional primary moment due to flange lateral bending. As shown below, $M_{ul}$ is not a real primary moment, but is a term that is combined with the actual primary moment to account for flange lateral bending stresses. Both $M_u$ and $M_{ul}$ are functions of the location of the cross section, given by $x$. The maximum value of $M_u + M_{ul}$ is taken as the combined demand for the unbraced segment $L_{b1}$. The notional primary moment due to flange lateral bending is defined as

$$M_{ul} = \frac{1}{3} f_{lu} S_{xc}$$
where $S_{sc}$ is the elastic section modulus about the major axis of the cross section to the compression flange and $f_{lu}$ is the flange lateral bending stress determined by

$$f_{lu} = \frac{|M_{lw}|}{t_f b_f^2 / 6}$$

where $M_{lw}$ is the flange lateral bending moment demand determined from an analysis of the CWG under vertical load using the methods discussed in Chapter 4 (e.g., Equation 4.8).

Using the same concept, the primary moment from nonlinear load deflection analyses is modified to define the LTB strength for CWGs that corresponds to $M_{u,\text{comb}}$. This LTB strength includes the effect of flange lateral bending and deflection under moment gradient bending. This combined LTB strength (moment capacity) $M_{FE,\text{comb}}$ is

$$M_{FE,\text{comb}} = M_{FE} + M_{FEL}$$

(5.3)

where $M_{FE}$ is the primary moment determined by nonlinear load deflection analysis and $M_{FEL}$ is the corresponding notional primary moment defined as

$$M_{FEL} = \frac{1}{3} f_{IFE} S_{sc}$$

(5.4)

where $f_{IFE}$ is the flange lateral bending stress determined by

$$f_{IFE} = \frac{|M_{IFE}|}{t_f b_f^2 / 6}$$

where $M_{IFE}$ is the compression flange lateral bending moment corresponding to the primary moment (see Figure 5.5) of magnitude $M_{FE}$. Both $M_{FE}$ and $M_{FEL}$ are functions of the location of the cross section, given by $x$. Therefore, to develop $M_{FE,\text{comb}}$ as the combined LTB strength, the variation with $x$ of the combined moment, and the contribution from the primary moment and flange lateral bending moment are considered, where $M_{FE,\text{comb}}(x)$, $M_{FE}(x)$, $M_{FEL}(x)$, $M_{IFE}(x)$, and $f_{IFE}(x)$ are symbols to represent the variables as they vary within the unbraced length. The maximum value of $M_{FE,\text{comb}}(x)$ within the unbraced segment $L_{u1}$ is taken as the combined LTB strength $M_{FE,\text{comb}}$.

To prevent LTB of CWGs under a moment gradient loading condition, a theoretical design comparison would be

$$M_{u,\text{comb}} \leq M_{FE,\text{comb}}$$

(5.5)

To simplify design practice, a nominal combined LTB strength $M_{n,\text{des}}$ will be developed so that

$$M_{n,\text{des}} \leq M_{FE,\text{comb}}$$

(5.6)

And in design practice, the following should be satisfied to prevent LTB.

$$M_{u,\text{comb}} \leq M_{n,\text{des}}$$

(5.7)
Together, Equation (5.6) and (5.7) enforce Equation (5.5).

Using the FE results presented earlier, the nominal combined LTB strength $M_{n\_des}$ will be developed based on Equation (5.6). Figure 5.16 shows how $M_{FE\_comb}(x)$ is determined from the primary moment $M_{FE}(x)$ and the notional primary moment $M_{FEL}(x)$ from FE results for a model of case NC88. $M_{FE}(x)$ was determined by a static analysis under load $p$ and $M_{end}$ (see Figure 5.5(a)) with magnitudes corresponding to their peak values from the nonlinear analysis (i.e., corresponds to $M = M_{cr}$ from the nonlinear analysis). $M_{FEL}(x)$ was determined from linear elastic static analysis results for the FE model of the full CWG girder by integrating the moment about the flange centroid from the axial normal stress at every cross section. Again the magnitude corresponds to $M = M_{cr}$. $M_{FE\_comb}(x)$ was determined from Equation (5.3) with $M_{FEL}(x)$ from $M_{IFE}(x)$ using Equation (5.4).

Figure 5.16 shows that $M_{FE\_comb}(x)$, $M_{FE}(x)$ and $M_{IFE}(x)$ are functions of position within the unbraced length. It can be seen that the maximum value of $M_{FE\_comb}(x)$ is at the right end. The results for the rest of the models for case NC88, with other unbraced lengths, are shown in Figure 5.17. Since the peak $M_{FE\_comb}(x)$ always occurs at the right end for unbraced segment $L_{b1}$, only the results for the three corrugations next to the right end are shown. It can be seen that the difference between $M_{FE\_comb}(x)$ and $M_{FE}(x)$ is small. The results also show that the flange lateral bending moment $M_{t\_des}$ from the proposed design formula, Equation (4.8), is conservative relative to $M_{IFE}(x)$ for all the models studied.

$M_{FE\_comb}$ for NC53 and NC93 was determined similarly. Figure 5.18 compares the peak values of $M_{FE\_comb}(x)$ and $M_{FE}(x)$, denoted as $M_{FE\_comb}$ and $M_{FE}$, for the three cases studied. It can be seen that the difference between $M_{FE\_comb}$ and $M_{FE}$ is small and the difference increases with a decrease in the lateral unbraced length. $M_{FE\_comb}$, which occurs at the right end, is taken as the LTB strength $M_{cr}$.

For conventional steel I-girders, under moment gradient bending, the LTB strength is usually calculated by modifying the LTB strength for uniform bending by the moment gradient factor. The same approach is used here for CWGs. The resulting LTB strength under moment gradient is

$$
\frac{M_{n\_des}}{M_p} = \frac{C_b}{\sqrt{1 + \lambda_M^{2\alpha}}} \leq 1.0
$$

where $C_b$ is the moment gradient factor defined by Equation (5.1). The remaining parameters are defined in Chapter 3 for the LTB of CWGs under uniform bending. The LTB strength of the three cases studied for unbraced segment $L_{b1}$ is determined using Equation (5.8) and the results are compared with the combined LTB strength.
$M_{FE\_comb}$ from the FE analyses in Figure 5.19. Remarkably, $M_{n\_des}$ from Equation (5.8) fits $M_{FE\_comb}$ very well even though it includes the effects of flange lateral bending, and $M_{n\_des}$ is conservative for nearly all the lateral unbraced lengths.

Finally, comparisons are made between the proposed LTB strength defined by Equation (5.8) and the design strength from the AASHTO LRFD Bridge Design Specifications (2004). Both the uniform torsional constant, $J$, for conventional steel I-girders and the modified uniform torsion constant, $J_{cu}$, for CWGs (discussed in Chapter 2) are used with the design formula from AASHTO LRFD Bridge Design Specifications (2004). The comparisons for the three selected cases are shown in Figure 5.20. It can be seen that no matter which torsional constant is used, results from the AASHTO LRFD Bridge Design Specifications will overestimate the LTB strength of CWGs for certain lateral unbraced lengths. The proposed nominal combined LTB strength $M_{n\_des}$ from Equation (5.8) compares much better with the nonlinear load deflection analysis results.

### 5.2.3 Finite Element Analysis Results for Segment $L_{b3}$

FE models of unbraced segment $L_{b3}$ were analyzed using both elastic buckling analyses and nonlinear load deflection analyses. Figure 5.21 shows the peak primary moment, $M_{FE\_b}$, at the middle of the unbraced segment (at mid span of the simply supported CWG with multiple unbraced lengths) versus the lateral unbraced length together with the elastic buckling analysis results for the FE models of cases NC14 and NC93. The peak primary moment from uniform bending nonlinear load deflection analyses are also shown. The moment gradient factor for unbraced segment $L_{b3}$ is very close to 1, as shown in Figure 5.4. The LTB behavior is expected to be very close to that under uniform bending. It can be seen from Figure 5.21 that for unbraced segment $L_{b3}$ the peak primary moments under moment gradient bending and from uniform bending compare very well. The peak primary moments under moment gradient bending for unbraced segment $L_{b3}$ are slightly larger than those from uniform bending, except for the models with two corrugations, for which the peak primary moments from uniform bending are larger. This may be attributed to the effects of shear which is present in the unbraced segment $L_{b3}$ under the moment gradient condition but not under uniform bending.

The combined LTB strength is also determined using Equation (5.3). Figure 5.22 shows how $M_{FE\_comb}$ is determined from the primary moment $M_{FE\_b}$ and the notional primary moment $M_{FE\_b}$ due to flange lateral bending for a model of case NC93. As discussed before, $M_{FE\_b\_l}(x)$ is determined from $M_{dFE\_l}(x)$ using Equation (5.4) and is combined with $M_{FE\_b\_l}(x)$ to determine $M_{FE\_comb\_l}(x)$ using Equation (5.3). The maximum value of $M_{FE\_comb\_l}(x)$ within the unbraced length is $M_{FE\_comb\_l}$, the combined LTB strength. It can be seen from Figure 5.22 that both $M_{FE\_comb\_l}(x)$ and
$M_{FE}(x)$ are symmetric about the mid span of the unbraced length. While the maximum $M_{FE}$ is at the mid span, the maximum $M_{FE,\text{comb}}$ is a quarter of a corrugation away from the mid span. The results also show that the flange lateral bending moment $M_{t,\text{des}}$ from the proposed design formula, Equation (4.8), is conservative relative to $M_{IFE}(x)$ for the model studied.

Figure 5.23 compares the combined LTB strength $M_{FE,\text{comb}}$ and the peak primary moment $M_{FE}$ for the two cases studied. It can be seen that the difference between $M_{FE,\text{comb}}$ and $M_{FE}$ is negligible, especially for case NC14 which has a smaller corrugation depth and corrugation length. The comparisons show that the effects of flange lateral bending are not significant for LTB of CWGs with practical dimensions under uniform bending and therefore, neglecting flange lateral bending in the studies presented in Chapter 3 is justifiable. The above comparisons show that the LTB of unbraced segment $L_{b3}$ can be treated as LTB under uniform bending, for which the effects of flange lateral bending moment can be neglected.

### 5.3 Comparisons of CWG and FWG LTB Strengths

In this section, the LTB performance of CWGs is compared with that of comparable conventional flat web I-girders (FWGs), as defined in Chapter 2. The comparison is made for both uniform bending and moment gradient bending for case NC88. For the moment gradient bending case studied, the girders are subjected to only a concentrated moment at the right end (see Figure 5.1). The elastic LTB strengths are determined from linear elastic buckling analyses and the inelastic LTB strengths are determined from nonlinear load deflection analyses. For the nonlinear load deflection analyses, the residual stresses are neglected but the realistic stress-strain model is used. Initial geometric imperfections are introduced as explained in Chapter 3.

For the FWGs, the web carries part of the primary bending moment. As a result, web bend buckling is possible. Since LTB is the subject of the study, web bend buckling was prevented by preventing cross section distortion in the FE model. At every cross section along the unbraced length, the rotations of all nodes on the cross section about the longitudinal axis are constrained to be equal. For a real I-girder, cross section distortion may be prevented only at the locations of transverse stiffeners. This constraint is applied only to the FWG FE models.

The FE analyses results are shown in Figure 5.24 where the peak primary moment for both the CWGs and FWGs are normalized by the plastic moment of the CWGs. For this investigation, the effects of vertical load induced flange lateral bending were not considered and only the peak primary moment is shown for the CWGs. It can be seen from Figure 5.24 that the peak moment for the CWGs from elastic buckling analyses are consistently larger than those of FWGs for both the uniform and the moment gradient bending, because CWGs have larger uniform torsion stiffness, as discussed in Chapter 2.

The nonlinear load deflection analyses show that for short unbraced lengths, the peak moments for the FWGs are larger than those of the CWGs, while for long
unbraced lengths, the peak moments for the CWGs are larger. This result is because for short unbraced lengths, the load deflection behavior is closer to that of in-plane bending and the peak moment approaches the plastic moment when the unbraced length decreases. Due to the contribution of the web, the plastic moment of the FWGs is higher than that of the CWGs. In addition, as shown in Figure 5.18, for short unbraced lengths, the combined LTB strength is greater than the peak primary moment for CWGs, and this combined LTB strength should be closer to the peak primary moment for the FWGs. For FE models with long unbraced lengths, the load deflection behavior is controlled by LTB. Since the CWGs have a larger uniform torsion stiffness, they have a greater peak moment.

5.4 Summary

This chapter investigated the LTB of CWGs under moment gradient bending. Due to the vertical load, the flanges of CWGs are displaced laterally, which adds flange lateral displacement to the initial geometric imperfection, resulting in a reduced LTB strength. To account for this effect in the nonlinear load deflection analyses, the initial geometric imperfections were applied in the direction in which the compression flange displaces when the CWG is under vertical load.

The effects of vertical load induced flange lateral bending are considered when the LTB strength of CWGs is defined using an approach similar to that used in the AASHTO LRFD Bridge Design Specifications (2004) to account for flange lateral bending. The effects of flange lateral bending turn out to be small for CWGs with practical dimensions, especially when the unbraced lengths are long or when the moment diagram is close to that of uniform bending.

A design formula for the LTB strength of CWGs under moment gradient bending was developed by modifying the LTB strength design formula for uniform bending with a moment gradient factor. The results from the proposed LTB strength design formula are conservative compared with the FE analyses results, for nearly all the cases studied. The results from the LTB strength design formula are closer to the FE analyses results than the results from the AASHTO LRFD Bridge Design Specifications (2004), which can significantly overestimate the LTB strength for certain lateral unbraced lengths.

The primary moment LTB strength of CWGs and FWGs were compared. CWGs have larger primary moment LTB strength for long unbraced lengths. For short unbraced lengths, the primary moment LTB strength of FWGs is larger.
Table 5.1 FE model loading for unbraced segment $L_{b1}$

(a) Loading for case NC53

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(c) Loading for case NC93

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Table 5.2 FE model loading for unbraced segment $L_{b3}$

(a) Loading for case NC14

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Figure 5.1 Loading used to study compression flange lateral displacement for NC88

Figure 5.2 Normalized compression flange lateral displacement for NC88
Figure 5.3 Effects of initial geometric imperfection directions

![Graph showing Effects of initial geometric imperfection directions.]

Figure 5.4 Selected lateral brace arrangement for simply supported CWG under uniformly distributed load

![Diagram showing selected lateral brace arrangement.]
(a) Unbraced segment $L_{b1}$

(b) Unbraced segment $L_{b3}$

Figure 5.5 FE models of isolated unbraced segments
Figure 5.6 Application of distributed load on flanges
Figure 5.7 Compression flange lateral displacement from the FE models of the full CWG and the isolated unbraced segments for NC88
(a) Unbraced segment $L_{b1}$

(b) Unbraced segment $L_{b3}$

Figure 5.8 Compression flange lateral displacement due to distributed load (and related primary shear)
Figure 5.9 Compression flange lateral displacement due to primary bending moment at end (and related primary shear)
Figure 5.10 Compression flange lateral displacement due to flange lateral bending moment at ends
Figure 5.11 Effects of vertical load induced flange lateral bending on compression flange initial geometric imperfection for NC88C8
(a) Primary moment versus right end rotation

(b) Primary moment versus top flange lateral rotation at the right end

Figure 5.12 Moment versus rotation for NC53 (L_{b1})
(a) Primary moment versus right end rotation

(b) Primary moment versus top flange lateral rotation at the right end

Figure 5.13 Moment versus rotation for NC88 (L_{01})
(a) Primary moment versus right end rotation

(b) Primary moment versus top flange lateral rotation at the right end

Figure 5.14 Moment versus rotation for NC93 (L_{01})
Figure 5.15 Primary moment versus lateral unbraced length ($L_{b1}$)
Figure 5.15 Primary moment versus lateral unbraced length ($L_{b1}$) (continued)
(a) Flange lateral bending moment from FE analysis and design formula

(b) Primary bending moment and combined bending moment from FE analysis

Figure 5.16 Combined LTB strength for NC88C6 (L_{b1})
(c) Primary and combined bending moment detail from FE analysis
Figure 5.16 Combined LTB strength for NC88C6 (L61) (continued)
Figure 5.17 Combined LTB strength of case NC88 from FE analysis ($L_{b1}$)
Figure 5.17 Combined LTB strength of case NC88 from FE analysis (Lb1) (continued)

(b) NC88C10
Figure 5.17 Combined LTB strength of case NC88 from FE analysis ($L_{b1}$) (continued)
(d) NC88C14
Figure 5.17 Combined LTB strength of case NC88 from FE analysis (Lb1) (continued)
Figure 5.17 Combined LTB strength of case NC88 from FE analysis (L_h1) (continued)
Figure 5.18 Combined LTB strength versus peak primary moment from FE analysis

(a) NC53

(b) NC88

Figure 5.18 Combined LTB strength versus peak primary moment from FE analysis ($L_{\text{b1}}$)
Figure 5.18 Combined LTB strength versus peak primary moment from FE analysis ($L_{b1}$) (continued)
Figure 5.19 Proposed combined LTB strength for moment gradient bending ($L_b$)

(a) NC53

(b) NC88
Figure 5.19 Proposed combined LTB strength for moment gradient bending ($L_b$)
(continued)
Figure 5.20 Comparison of LTB strength design formulas for moment gradient bending ($L_b$)

(a) NC53

(b) NC88
Figure 5.20 Comparison of LTB strength design formulas for moment gradient bending ($L_{b1}$) (continued)

(c) NC93

Nonlinear analysis
Proposed formula
AASHTO (J)
AASHTO ($J_{cw}$)
Figure 5.21 Primary moment versus lateral unbraced length ($L_b$)

(a) NC14

(b) NC93

276
(a) Flange lateral bending moment from FE analysis and design formula

(b) Primary bending moment and combined bending moment from FE analysis

Figure 5.22 Combined LTB strength for NC93C6 ($L_{b3}$)
(c) Primary and combined bending moment detail from FE analysis
Figure 5.22 Combined LTB strength for NC93C6 (Lₐ₃) (continued)
Figure 5.23 Combined LTB strength versus peak primary moment ($L_{b3}$)

(a) NC14

(b) NC93
Figure 5.24 Comparisons of the LTB strength of CWGs and FWGs (NC88)
6 Summary, Findings, Conclusions, Contributions, and Future Work

6.1 Summary

This report presents research on the lateral torsional buckling (LTB) strength of steel corrugated web I-girders (CWGs) under both uniform and moment gradient bending. The objectives of this research were as follows:

- To investigate the uniform torsion of CWGs and to develop an analytical model, which is capable of predicting the torsional stiffness of CWGs.
- To investigate the LTB behavior of CWGs under uniform bending moment.
- To investigate flange lateral bending of CWGs under vertical load.
- To investigate the LTB behavior of CWGs under moment gradient bending.
- To develop design formulas for the nominal LTB strength of CWGs, which can be used to design CWGs for highway bridges.

To accomplish these objectives, detailed finite element (FE) models of CWGs were developed. These models consider all the major factors that determine the LTB strength of CWGs. These factors include the lateral unbraced length, initial geometric imperfections, steel stress-strain behavior, and residual stresses. Detailed studies of these factors were made. In addition, to enable the torsional stiffness of CWGs to be easily and accurately estimated, a detailed study of the behavior of CWGs under uniform torsion was made. The FE models were developed and analyzed using the general purpose FE analysis package ABAQUS versions 6.3 and 6.5. The CWGs considered in these studies have two identical flanges with a trapezoidally corrugated web.

With regard to the uniform torsion of CWGs, the following main tasks have been completed:

1. FE models of corrugated web girders and comparable conventional flat web I-girders (FWGs) were developed. A FE mesh was selected through a mesh convergence test.
2. The uniform torsion of a prototype CWG was investigated through a torsional deformation study and a strain energy study. A uniform torsion resistance mechanism, corrugation torsion, was defined.
3. Both a kinematic analysis and a simple static analysis based on the approach of Lindner and Aschinger (1990) were used to explain the corrugation torsion.
4. Corrugation torsion internal force distributions were studied using FE analyses. Static equilibrium formulations were developed.
5. A corrugation torsion model was proposed based on the results of FE analyses.
6. A static solution to the corrugation torsion was developed and internal forces for both the web and the flanges were derived.
7. A corrugation torsion stiffness was derived using an energy approach, which depends on an undetermined parameter $\lambda$. A solution for the corrugation torsion model was selected based on comparisons of the calculated flange internal forces and the FE analysis results.
8. A large number of CWGs with practical dimensions were selected. The reaction torque under an imposed twist for each CWG and the comparable FWG were determined from FE analyses. The parameter $\lambda$ was then determined for each case. A regression equation was developed for $\lambda$. The results were evaluated by comparing the calculated reaction torque and corresponding internal forces with the FE analysis results.

9. A regression equation was also determined for the ratio of the CWG total reaction torque $T$ to the FWG reaction torque $T_f$, $T/T_f$.

With regard to the LTB of CWGs under uniform bending, the following main tasks have been completed:

1. The LTB strength of CWGs under uniform bending moment was studied and a formula for the nominal LTB capacity was developed based on results from FE analyses.
2. The elastic LTB strength was determined by linear elastic buckling analysis (using the Buckle command in ABAQUS v6.3). The inelastic LTB strength was determined from an incremental nonlinear inelastic load deflection analysis (using the modified Riks method available in ABAQUS v6.3).
3. FE models were developed for the nonlinear analyses. Detailed studies were made to incorporate initial geometric imperfections, realistic steel stress-strain models, and residual stresses into the FE models. A special FE mesh was developed for CWGs to consider residual stresses and the corrugated web geometry.
4. Parameter studies were conducted to study the effects of initial geometric imperfections, steel stress-strain behavior and residual stresses on the LTB strength of CWGs. Based on the studies, the final finite element modeling approach was determined.
5. FE models of eight selected CWG cases were developed. The LTB strengths of these eight cases were determined for various unbraced lengths from nonlinear analyses of these FE models. Based on these results a nominal LTB capacity design formula was developed and presented.
6. Results from the proposed design formula were compared with the nominal LTB capacities from both the AASHTO LRFD Bridge Design Specifications (2004) and the German DIN 18800 Specifications.

With regard to flange lateral bending under vertical load, the following main tasks have been completed:

1. The fictitious load approach (Abbas 2003) was used to study the primary shear induced flange lateral bending moment in CWGs in the presence of intermediate braces.
2. A comparison of the flange lateral displacements from the fictitious load approach and FE analysis of a CWG was made.
3. Flange lateral bending under only primary bending was also investigated and a simple formula is proposed to calculate the flange lateral bending moment due to the primary bending moment.
4. A modified design formula was proposed which considers the effects of both primary shear and primary bending moment induced flange lateral bending moment and compared with FE analysis results.

With regard to the LTB of CWGs under moment gradient bending, the following main tasks have been completed:

1. The interaction of the vertical load induced flange lateral displacement with the initial geometric imperfection and its effects on the LTB strength of a CWG were studied.
2. FE models for LTB under moment gradient bending were developed based on simply supported CWGs with multiple intermediate lateral braces under uniformly distributed load.
3. The LTB strength of CWGs under two moment gradient conditions was studied in detail considering the effects of vertical load induced flange lateral bending moment.
4. A nominal LTB capacity design formula for CWGs under moment gradient bending was developed and presented.
5. Results from the proposed design formula were compared with the nominal LTB capacities from the AASHTO LRFD Bridge Design Specifications (2004).
6. The LTB strength of CWGs and FWGs under both uniform and moment gradient primary bending were compared.

6.2 Findings

Based on the research presented in this report, the main findings for each of the four research areas are:

**Uniform torsion of corrugated web girders:**

- The kinematics of CWGs under uniform torsion causes rotation about the CWG transverse axis of the longitudinal web folds, which is restrained by the flange bending stiffness. This uniform torsion resistance mechanism was named corrugation torsion.
- The deformations and stresses related to corrugation torsion repeat themselves from corrugation to corrugation.
- Corrugation torsion related normal forces, transverse shear forces and lateral bending moments on the two flanges are equal but opposite in sign, while the vertical shear forces and the bending moments about the flange weak axis are the same. The vertical shear force on the web is twice as large as the vertical shear force on either flange but opposite in sign.
- Calculation of the internal forces that develop during corrugation torsion is a highly statically indeterminate problem. The results from the corrugation torsion model are able to accurately predict the corrugation torsion stiffness and the flange bending moment about its weak axis.

**Lateral Torsional Buckling Under Uniform Bending:**

- Due to the corrugations, compression flange lateral bending is often accompanied by flange local plate bending. The LTB results presented in the
report were not affected by flange local plate bending since compact flanges were used.

- Parameter studies using FE models show that as the initial geometric imperfection amplitude increases, the LTB strength decreases. The two steel stress-strain models used in these studies did not cause any significant difference in the LTB strength. The LTB strength is reduced due to the presence of residual stresses, especially for intermediate lateral unbraced lengths.

- The most important factors that affect the lateral torsional buckling strength of corrugated web I-girders are lateral unbraced length, initial geometric imperfection, and residual stresses.

- Results from the proposed nominal LTB capacity design formula agree with the FE analysis results very well for all lateral unbraced lengths studied. Design formulas from the DIN 18800 part 2 overestimate the LTB strength of CWGs with small lateral unbraced lengths. Design formulas from the AASHTO LRFD Bridge Design Specifications (2004) also overestimate the LTB strength of girders with small and intermediate lateral unbraced lengths.

Flange Lateral Bending under Vertical Load:

- The presence of intermediate braces reduces the flange lateral displacement induced by vertical load. The brace forces change the overall pattern of the flange lateral bending moment over the CWG span while the local variation of the flange lateral bending moment does not change.

- The fictitious load approach (Abbas 2003), intended to determine flange lateral bending induced by primary shear, underestimates the flange lateral bending moment for long CWGs, but provides good results for short CWGs.

- The flange lateral displacement under primary shear normalized by the span length decreases with an increase in span length.

- Due to the deformation incompatibility of the flange and the corrugated web under axial extension or axial compression, flange lateral bending is induced by primary bending moment alone.

- The flange lateral bending moment induced by the primary bending moment is independent of the span length. It is proportional to primary bending moment and is inversely proportional to the girder depth.

- The flange lateral bending moment induced by primary bending moment achieves its maximum magnitude at the centers of the longitudinal folds and is zero at the centers of the inclined folds.

Lateral Torsional Buckling under Moment Gradient Bending:

- To determine the minimum LTB strength for a given CWG and unbraced length from FE analysis, the initial geometric imperfection should be applied so that the lateral displacements of the compression flange due to the initial geometric imperfection and due to the flange lateral bending under vertical load are in the same direction.
• For a lateral unbraced segment with a large moment gradient factor (a large primary shear), the flange lateral displacement due to vertical load is significant compared to that due to initial geometric imperfection.
• For a lateral unbraced segment with a small moment gradient factor (little or no primary shear), the flange lateral displacement due to vertical load is negligible compared to that due to initial geometric imperfection.
• Nonlinear analysis shows that the peak primary moment at LTB under moment gradient bending is significantly larger than the peak primary moment at LTB under uniform bending for an unbraced segment with a large moment gradient factor.
• For the practical cases studied, the effects of vertical load induced flange lateral bending moment on the peak primary moment is small (less than 4%), especially for girders with a long unbraced length.
• The nominal LTB capacity design formula proposed by this research accounts for both primary moment and vertical load induced flange lateral bending moment. The proposed nominal LTB capacity design formula is conservative compared to FE analysis results for nearly all of the CWG cases and lateral unbraced lengths considered.
• Design formulas from the AASHTO LRFD Bridge Design Specifications (2004) overestimate the LTB strength of CWGs with intermediate lateral unbraced lengths.
• For a lateral unbraced segment with a moment gradient factor very close to unity, the peak primary moment is very close to the result for uniform bending except for a very short unbraced length for which the ultimate moment is slightly reduced due to the presence of vertical shear.
• For the practical cases studied, when the moment gradient factor is very close to unity, the effects of vertical load induced flange lateral bending moment on the peak primary moment are small (less than 1%) and negligible.

6.3 Conclusions

Based on the research presented in this report, the following conclusions are drawn:

• Under uniform torsion, a corrugated web girder (CWG) is stiffer than a comparable conventional flat web I-girder (FWG). The total torsional stiffness of CWGs under uniform torsion is equal to the sum of the St. Venant torsion stiffness (which is similar to that of a FWG) and the corrugation torsion stiffness.
• Relative to conventional I-girders, the warping torsion stiffness and lateral bending stiffness of CWGs appear to be nearly unchanged.
• Flange lateral bending is introduced under both primary bending moment and primary shear. A flange lateral bending moment design formula proposed in the report is conservative compared with the FE analysis results.
• Under both uniform bending and moment gradient bending, the elastic LTB strength of CWGs is greater than that of comparable conventional I-girders due to the increased uniform torsion stiffness.
• The inelastic LTB strength of CWGs is less than that of comparable conventional I-girders without section distortion, because the conventional I-girder has a larger in-plane bending capacity and a simpler stress state in the flanges.
• The proposed nominal LTB capacity design formula agrees with the FE analysis results very well in both elastic and inelastic LTB under uniform bending and under moment gradient bending when the moment gradient factor is included.

6.4 Contributions
This research has made the following contributions to knowledge about the behavior of CWGs under torsion and flexure:
• Provided an improved understanding of CWG behavior under uniform torsion including kinematics and statics.
• Developed a corrugation torsion model based on FE analysis results.
• Developed a method to consider residual stresses in FE models of CWGs.
• Provided an improved understanding of the flange lateral bending induced by vertical load, including primary shear and primary moment induced flange lateral bending.
• Provide an improved understanding of the relative LTB strengths of CWGs and FWGs.
• Developed design formulas for the nominal LTB capacity of CWGs.

6.5 Recommendations for Future Work
The work presented in this report is based on finite element analysis. Experimental work was not feasible within the resources available to the research project. To verify the findings and conclusions in this report, experimental work should be conducted. The residual stresses used in the finite element model are derived from available information for conventional flat web I-girders. Residual stresses in corrugated web girders should be measured to verify the assumed residual stresses and studies should be conducted to document the effects of these residual stresses on the LTB strength of corrugated web girders.
It has been shown in this report that flange lateral bending is induced by primary bending moment, and a simplified model has been proposed to determine the maximum flange lateral bending moment induced by primary bending moment. To develop more accurate models, a more detailed study should be made.
The corrugated web girders investigated in this report have identical top and bottom flanges. Corrugated web girders used for highway bridges may have different top and bottom flanges. The behavior of such girders should be investigated. Curved corrugated web girders and corrugated web girders with variable web depth also need to be investigated in the future.
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